TRIGONOMETRY
SLIDE RULE
MECHANICS

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By

I.C.S. STAFF

TRIGONOMETRY AND GRAPHS
THE SLIDE RULE
MECHANICS AND MACHINE ELEMENTS

145

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TRIGONOMETRY AND GRAPHS

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TRIGONOMETRY

FUNCTIONS OF ANGLES

GENERAL REMARKS

1. Parts of a Triangle.—A triangle may be considered as being made up of six parts, namely, three sides and three angles. The lengths of the sides are expressed in inches, feet, etc., and the angles in degrees and subdivisions of a degree. If three of the six parts of a triangle are known, one of them being the length of a side, the other three parts may be calculated. For example, if the lengths of the three sides are known, the sizes of the three angles can be found. If two angles and the length of one side are known, the other angle and the lengths of the two remaining sides can be calculated. If two sides and one angle are known, the other side and the two remaining angles can be found. The process of calculating the values of the unknown sides or angles, having given the values of three of the six parts of the triangle, is called the **solution of the triangle.**

2. Trigonometry.—There is one branch of mathematics that relates to the solution of triangles; accordingly, it is called **trigonometry.** This term is derived from two Greek words: *trigonon,* meaning *triangle;* and *metron,* meaning *measure.* Thus, trigonometry is simply the branch of mathematics that

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deals with the measurement and calculation of the parts of triangles.

3. Functions.—The work of calculating the values of the sides and angles of triangles is made fairly easy by the use of what are called *trigonometric functions of angles.* To understand this expression it is necessary, first of all, to know what is meant by a function. A **function** of any quantity is simply a second quantity, the value of which depends on the value of the first quantity. For example, the circumference of a circle always has a length equal to 3.1416 times the diameter; that is, the value of the circumference depends on the value of the diameter. Therefore, the circumference of a circle is a function of the diameter. On the other hand, the diameter of a circle is always equal to the circumference divided by 3.1416; hence, the diameter is a function of the circumference. The distance an electric car travels depends on the speed and the time; therefore, the distance traveled is a function of the time and speed. The resistance of an electric circuit is a function of the length of the circuit, because the resistance depends directly on the length.

4. Value of a Knowledge of Trigonometry.—A fair working knowledge of the principles of trigonometry is valuable to any one engaged in engineering work, but it is extremely important to the student of electrical engineering because it forms the basis for the solution of many problems in connection with alternating currents; in fact, the solution of many of these problems would be almost impossible for the average student if he were not able to use trigonometry. Besides, there are many other classes of problems to which it may be applied. For example, the power-plant man may use it to determine the height of a chimney. A foreman who is putting up wires may use it to find the height of a tree or of a standing pole. A workman fitting a piece of timber to act as a slanting brace may find by trigonometry the angles to which the ends of the piece must be cut. The variety of practical problems that may be solved by the principles of trigonometry is almost endless.

TRIGONOMETRIC FUNCTIONS

5. Functions of an Angle.—An angle—by which is meant the amount of opening between two straight lines that meet at a point—also possesses certain functions. To illustrate, take the case of the right triangle $A\,B\,C$, Fig. 1, which has a right angle at C. The sides of this triangle will be designated by a, b, and c, as shown, a being the side opposite the angle A, b that opposite the angle B, and c that opposite the right angle C. If the angle A were made larger, a would have to be longer; therefore, as the length a depends on the size of the angle A, it is a function of the angle A. At the same time, c would become longer, so that it also is a function of the angle A. Carrying this idea further, the size of the angle B likewise controls the relative lengths of the sides. It is impossible to change the size of an angle without causing changes in

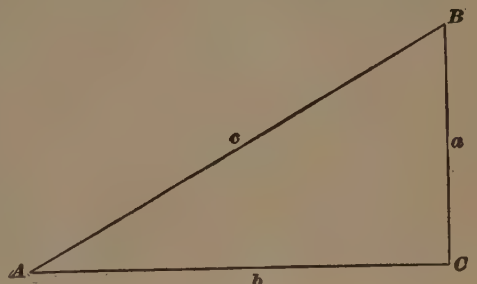

FIG. 1

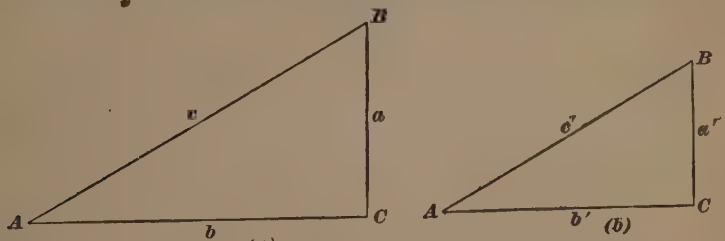

FIG. 2

the lengths of the sides, so long as C remains a right angle. Therefore, the sides of a right triangle are functions of the angles.

6. In Fig. 2 (a), suppose that the sides b and c make an angle of 30° with each other, or, in other words, that the

angle A measures 30°; also, suppose that C is a right angle. Then, the side c opposite the right angle is the hypotenuse. Now, with reference to the angle A, the side a is the *side opposite* that angle and the side b is the *side adjacent* (or lying next) to that angle. If the hypotenuse c has a length of 2 inches, then, on measuring the side a, it will be found to be 1 inch long. The ratio of a to c will then be 1 : 2; that is, the ratio between the side opposite the angle A and the hypotenuse, when the angle A is 30°, has a value of $\frac{1}{2}$, or $\frac{a}{c}=\frac{1}{2}$.

7. Next, suppose that another right triangle is drawn, as in Fig. 2 (b), having an angle A of 30°, as before, but with a hypotenuse c' only $1\frac{1}{2}$ inches long. If the length of the side a' is measured, it will be found to be $\frac{3}{4}$ in., or just half as long as c'. Then, considering the 30° angle A, as before, the ratio between the side opposite and the hypotenuse is $\frac{3}{4} : 1\frac{1}{2}$, or $\frac{a'}{c'}=\frac{\frac{3}{4}}{1\frac{1}{2}}=\frac{1}{2}$. This value, it will be seen, is exactly the same as was obtained for the ratio between the side opposite and the hypotenuse in the case of the angle A in (a). Moreover, if other right triangles are drawn, of any size whatever, with one angle measuring 30°, the ratio between the side opposite that angle and the hypotenuse will always be 1 : 2, or have a value of $\frac{1}{2}$. This leads to the important conclusion that, so long as the angle A remains 30°, the side opposite will always be half as long as the hypotenuse; that is, the ratios $\frac{a}{c}$ and $\frac{a'}{c'}$ will always be $\frac{1}{2}$.

8. In the case of the triangles shown in Fig. 2, other ratios may be formed, besides the one just mentioned. For example, a ratio may be formed between the side a opposite the angle A and the side b adjacent to it. Furthermore, the values of this ratio for the two triangles in (a) and (b) are equal; that is, $\frac{a}{b}$ in (a) is equal to $\frac{a'}{b'}$ in (b), even though the lengths of a and b are different from the lengths of a' and b'. This statement may easily be proved, as follows: According to the principles of

geometry, the sum of the three angles of every triangle is equal to 180°. In each of the two cases shown, the angle A is 30° and the angle C is a right angle, or 90°, the sum of the two, A and C, being 120°; therefore, the angle B in each must be $180° - 120° = 60°$. This means that the three angles of one are equal to the three angles of the other and so the two triangles are similar. Now, it is a law of geometry that if two figures are similar, their corresponding sides are proportional; so, $a : b = a' : b'$, or $\frac{a}{b} = \frac{a'}{b'}$. In this proportion, the first ratio refers to the sides of the triangle in (a) and the second ratio to the triangle in (b). This is the statement that was to be proved and shows that, so long as the angle A is unchanged in size, the ratio between the side opposite and the side adjacent is always the same, no matter how large or how small the right triangle may be.

9. In the preceding articles, only two different ratios have been considered—that of the side opposite and the hypotenuse and that of the side opposite and the side adjacent. There are several other ratios that may be formed. Every triangle has three sides, and therefore it is possible to form a ratio between one side and each of the other two remaining sides, or six ratios in all. In the triangle in Fig. 2 (a), these six ratios are $\frac{a}{b}, \frac{a}{c}, \frac{b}{a}, \frac{b}{c}, \frac{c}{a}$, and $\frac{c}{b}$. So long as the angles of the right triangle remain unchanged, these various ratios will not alter in value; and as they are formed by using the lengths of the sides a, b, and c, they are functions of the angles, because their values change when the sizes of the angles are changed.

10. Trigonometric Functions of Angles.—The six ratios stated in the preceding article form what are called the **trigonometric functions of an angle,** and each one is given a specific name so that it may be distinguished from the others. The ratio between the side opposite and the hypotenuse is called the **sine.** The ratio between the side adjacent and the hypotenuse is called the **cosine.** The ratio between the side opposite and the side adjacent is called the **tangent;** and the

ratio between the side adjacent and the side opposite is called the **cotangent.** These are the functions most commonly used and are the only ones that will be referred to in the later parts of this Section. The two remaining ratios, or functions, are seldom used. They are: the **secant,** which is the ratio of the hypotenuse to the side adjacent; and the **cosecant,** which is the ratio of the hypotenuse to the side opposite.

11. When writing the names of the various ratios, it is customary to use abbreviations, so as to save time and space; thus, the word *sine* is abbreviated **sin,** *cosine* is abbreviated **cos,** *tangent* is abbreviated **tan,** and *cotangent* is abbreviated **cot.** Then, sin 26° is read *sine of 26°;* cos 15° is read *cosine of 15°;* tan 3° 27′ is read *tangent of 3° 27′.* It should be observed that these abbreviations are *not* followed by periods. It should also be observed that the word *of* is *not* used in connection with the abbreviations. Thus, although *sin* is the abbreviated form of *sine,* it stands for the two words *sine of,* in expressions like sin 21°, sin 12°, etc. Similarly *tan, cos,* and *cot* take the place of *tangent of, cosine of,* and *cotangent of,* although they are the abbreviations of the words *tangent, cosine,* and *cotangent,* respectively.

The ratios for the common functions of the angle A, Fig. 1, are as follows:

Ratio	Function	Abbreviation
$\frac{\text{Side opposite}}{\text{Hypotenuse}} = \frac{a}{c}$	sine of A	$\sin A$
$\frac{\text{Side adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$	cosine of A	$\cos A$
$\frac{\text{Side opposite}}{\text{Side adjacent}} = \frac{a}{b}$	tangent of A	$\tan A$
$\frac{\text{Side adjacent}}{\text{Side opposite}} = \frac{b}{a}$	cotangent of A	$\cot A$

12. It should be noted carefully that the sine of the angle A, Fig. 1, is the ratio $\frac{a}{c}$ and that this particular ratio is correct only for sin A. If the angle B is considered, the side opposite the angle is b, the side adjacent is a, and the hypot-

enuse is c as before. Therefore, considering the angle B, the four main functions are as follows:

$$\sin B = \frac{b}{c} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\cos B = \frac{a}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\tan B = \frac{b}{a} = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\cot B = \frac{a}{b} = \frac{\text{side adjacent}}{\text{side opposite}}$$

The point to be kept in mind is that the sine is always the ratio between the side opposite and the hypotenuse; the cosine is always the ratio between the side adjacent and the hypotenuse; and so on for each of the other functions.

13. It will be observed that the ratios given in Arts. **11** and **12** are alike, but that they differ in their order and in their meanings. For example, Art. **11** shows that $\frac{a}{c} = \sin A$ and Art. **12** shows that $\frac{a}{c} = \cos B$. The ratio $\frac{a}{c}$ is the same in both cases and its value must be the same in both, because the sides a and c remain unchanged in length; therefore, it follows that $\sin A = \cos B$. Similarly, $\frac{b}{c}$ is equal to $\cos A$, but it is also equal to $\sin B$; therefore $\cos A = \sin B$. Again, $\frac{a}{b} = \tan A$ and $\frac{a}{b} = \cot B$; therefore, $\tan A = \cot B$. And finally, $\frac{b}{a} = \cot A$ and also $\frac{b}{a} = \tan B$; therefore, $\cot A = \tan B$. These relations are very important.

14. Deriving Functions of 30° and 60°.—An equilateral triangle, or a triangle with equal sides and equal angles, is shown in Fig. 3 (a). If a perpendicular line $B\,C$ is drawn from B to the base $A\,D$, it will make two angles of 90° with the base and will divide the base into two equal parts. Suppose that the sides are 2 inches long; then, each part of the base

is 1 inch long. The three angles of the triangle must be 60° each, because their sum must be 180°. The line $B\,C$ will divide the angle B into two equal parts, each 30°. Now, suppose that the left-hand triangle $A\,B\,C$ is set off by itself, as in (b), and

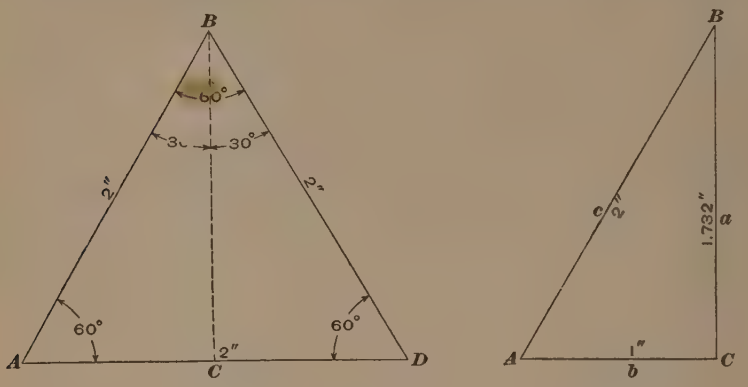

FIG. 3

that the functions of the angles A and B are to be found. It is first necessary to find the length of the side a. As the triangle is a right triangle,

$$a=\sqrt{c^2-b^2}=\sqrt{2^2-1^2}=\sqrt{4-1}=\sqrt{3}=1.732 \text{ inches}$$

Considering the angle B, which is 30°, the side opposite is b and the side adjacent is a. Therefore,

$$\sin 30°=\sin B=\frac{b}{c}=\frac{1}{2}=.500$$

$$\cos 30°=\cos B=\frac{a}{c}=\frac{1.732}{2}=.866$$

$$\tan 30°=\tan B=\frac{b}{a}=\frac{1}{1.732}=.577$$

$$\cot 30°=\cot B=\frac{a}{b}=\frac{1.732}{1}=1.732$$

Considering the angle A, which is 60°, the side opposite is a and the side adjacent is b. Therefore,

$$\sin 60°=\sin A=\frac{a}{c}=\frac{1.732}{2}=.866$$

$$\cos 60° = \cos A = \frac{b}{c} = \frac{1}{2} = .500$$

$$\tan 60° = \tan A = \frac{a}{b} = \frac{1.732}{1} = 1.732$$

$$\cot 60° = \cot A = \frac{b}{a} = \frac{1}{1.732} = .577$$

15. Deriving Functions of 45°.—The functions of 30° and 60° are used more frequently than any others, particularly by electrical engineers, but the functions of 45° are also required very often. The triangle shown in Fig. 4 is a right triangle with each acute angle 45°. Let each side be 1 inch long; then, the length of the hypotenuse will be $\sqrt{1^2+1^2} = \sqrt{2} = 1.414$ inches. The side opposite and the side adjacent are the same for each angle A and B; therefore, the values of the functions of A are exactly the same as those of B. Thus, if the angle A is considered,

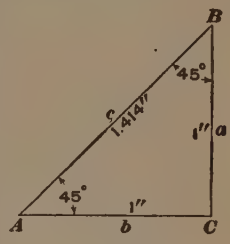

FIG. 4

$$\sin 45° = \sin A = \frac{1}{1.414} = .707$$

$$\cos 45° = \cos A = \frac{1}{1.414} = .707$$

$$\tan 45° = \tan A = \frac{1}{1} = 1.000$$

$$\cot 45° = \cot A = \frac{1}{1} = 1.000$$

Similarly, if the angle B is considered,

$$\sin 45° = \sin B = \frac{1}{1.414} = .707$$

$$\cos 45° = \cos B = \frac{1}{1.414} = .707$$

$$\tan 45° = \tan B = \frac{1}{1} = 1.000$$

$$\cot 45° = \cot B = \frac{1}{1} = 1.000$$

From the foregoing statements it may be seen that $\sin 45° = \cos 45°$, and therefore $\sin A = \sin B = \cos A = \cos B$. Similarly, $\tan 45° = \cot 45°$ and therefore $\tan A = \tan B = \cot A = \cot B$. These relations are true, however, *only* when the angles A and B are equal and each is 45°.

16. Functions of Complementary Angles.—It has already been stated that the sum of the three angles of a triangle is 180°. As the angle C, Fig. 1, is 90°, the sum of the two remaining angles A and B must be $180° - 90° = 90°$; that is, $A + B = 90°$. This is true of every right triangle, no matter what its size or shape may be. In the particular case shown, A is 30° and B is 60°, their sum being 90°. Now, when any two angles have such values that, when added together, their sum is 90°, they are said to be *complementary angles*, and each is called the **complement** of the other. Thus, A and B are complements of each other, because their sum is 90°; so are 42° and 48°, 16° and 74°, 25° and 65°, and many others. If the complement of any angle is to be found, the given angle is subtracted from 90° and the remainder is the complement. Thus, the complement of 30° is $90° - 30° = 60°$; the complement of 39° 45′ is $90° - 39°\ 45′ = 50°\ 15′$; and the complement of 72° 12′ is $90° - 72°\ 12′ = 17°\ 48′$.

17. As the angle A, Fig. 3 (b), is the complement of the angle B and the angle B is the complement of the angle A, the relations between their functions, as given in Art. **14**, may now be stated as follows:

$$\sin A = \cos B = \cos (90° - A)$$
$$\cos A = \sin B = \sin (90° - A)$$
$$\tan A = \cot B = \cot (90° - A)$$
$$\cot A = \tan B = \tan (90° - A)$$
$$\sin B = \cos A = \cos (90° - B)$$
$$\cos B = \sin A = \sin (90° - B)$$
$$\tan B = \cot A = \cot (90° - B)$$
$$\cot B = \tan A = \tan (90° - B)$$

These facts may be stated in a general way, as follows:

1 The sine of any angle is equal to the cosine of its complement.

2. The cosine of any angle is equal to the sine of its complement.

3. The tangent of any angle is equal to the cotangent of its complement.

4. The cotangent of any angle is equal to the tangent of its complement.

To illustrate these points clearly, suppose that, in Fig. 1, the sides a, b, and c have lengths of 1.25, 2.165, and 2.5 inches, respectively. Then,

$$\sin A = \cos B = \frac{a}{c} = \frac{1.25}{2.5} = .5 \qquad \tan A = \cot B = \frac{a}{b} = \frac{1.25}{2.165} = .577$$

$$\cos A = \sin B = \frac{b}{c} = \frac{2.165}{2.5} = .866 \qquad \cot A = \tan B = \frac{b}{a} = \frac{2.165}{1.25} = 1.732$$

18. Functions of Supplementary Angles.—If the sum of two angles is 180°, the angles are *supplementary* and each is said to be the **supplement** of the other; thus, 106° and 74° are supplementary angles, because $106° + 74° = 180°$. For the same reason, 120° and 60° are supplementary, as are 89° and 91°, 27° and 153°, and many other combinations. To find the supplement of an angle, it is simply necessary to subtract that angle from 180°. Thus, the supplement of 52° 24′ is $180° - 52°\ 24' = 127°\ 36'$. The values of the trigonometric functions of two supplementary angles are numerically equal, each to each; that is, a function of an angle is numerically equal to the same function of its supplement. Therefore, considering any angle, as A, the following relations are true, so far as the numerical values of the functions are concerned:

$$\sin A = \sin (180° - A) \qquad \tan A = \tan (180° - A)$$
$$\cos A = \cos (180° - A) \qquad \cot A = \cot (180° - A)$$

Therefore, if the angle given is greater than 90° and less than 180°, and some function of it is required, subtract it from 180° and find the same function of the remaining angle.

19. Table of Natural Trigonometric Functions. From what has already been stated with regard to the trigonometric functions of angles, it is seen that for a given angle there is one unchanging value of its sine, a single value for its

cosine, and so on for each of the other functions. It is therefore possible to arrange a table in which are given the various values of the functions corresponding to the different values of angles. Such a table will be found at the end of this Section. It gives the values of the sines, cosines, tangents, and cotangents of all angles from 0° to 90°, to five decimal places. These trigonometric functions are known as the natural functions of the angles, and so the values are called the natural sines, natural cosines, etc. This table is useful in solving many kinds of engineering problems, as will be shown later.

20. The table at the end of this Section is divided into two parts. The first nine pages of the table contain the values of the natural sines and cosines and the remaining nine pages contain the values of the natural tangents and cotangents. For all ordinary problems it will be found sufficiently accurate if the value of the angle is taken to the nearest minute. The table is therefore arranged so as to show the values of the functions for all angles from 0° to 90°, inclusive, increasing by 1′. All angles expressed in whole numbers of degrees from 0° to 44° will be found in the rows at the tops of the pages, the minutes being given in the first column of the page, reading downwards. The values of the functions are given in the columns headed by the names of the functions. For whole degrees from 45° to 89° it is necessary to look along the bottoms of the pages, the minutes being given in the right-hand column, reading *upwards*. As 90° is equal to 89° 60′, the values of the functions of 90° will be found on the first and tenth pages of the table, in the first line and in the second and third columns, as will be explained later.

It will be noticed that the columns headed Sine at the top are marked Cosine at the bottom, and those marked Cosine at the top are marked Sine at the bottom; similarly, the terms Tang and Cotang are interchanged at top and bottom of each column, in the latter half of the table. The reason for this will be understood from what is said in Art. **17.** The angle corresponding to any particular value in the table, reading the value of the angle from the *top*, is the complement of

the angle corresponding to that value, reading the value of the angle from the *bottom*. That is why the table is read downwards from 0° to 45° and upwards from 45° to 90°.

21. The table gives the values of the functions of angles expressed in degrees and minutes only. So, if the value of an angle is given in degrees, minutes, and seconds, the angle should be taken to the nearest number of minutes and the function corresponding to this latter value should be found. As there are 60 seconds in 1 minute, the middle point is half of 60, or 30; so, if an angle contains less than 30″, ignore the number of seconds, but if it contains 30″ or more than 30″, drop the number of seconds and add 1′ to the number of minutes in the angle. These points may be illustrated by examples. For instance, consider the angle 12° 27′ 45″. As 45″ is greater than 30″, add 1′ to the number of minutes in the angle, which will give 12° 28′; that is, 12° 27′ 45″ is 12° 28′, approximately. Similarly, 65° 48′ 22″ is 65° 48′, approximately, as 22″ is less than 30″ and is ignored; 7° 18′ 32″ becomes 7° 19′, as 32″ is more than 30″; 47° 52′ 8″ is taken as 47° 52′; 29° 6′ 51″ is taken as 29° 7′; 35′ 40″ is taken as 36′; 4° 30′ 30″ is taken as 4° 31′; and so on.

22. Limits of the Natural Functions.—Between 0° and 90° the sine increases from a value of zero to a value of 1, while the cosine at the same time decreases from a value of 1 to a value of zero. The tangent of an angle of 0° is zero, from which value it increases as the angle increases, until its value for an angle of 90° is infinity. This word *infinity* simply means a value so great that it cannot be measured. The cotangent of an angle of 0° is infinity, and it decreases as the angle increases, until its value for an angle of 90° is zero.

23. Inverse Functions.—In various books and magazines will be found expressions like $A = \sin^{-1} .86603$, $B = \tan^{-1} .47382$, etc. These are trigonometric expressions and are called **inverse functions.** The first one is read *A equals the angle whose sine is .86603* and the second is read *B equals the angle whose tangent is .47382*. The only reason for using such an expression is that it saves writing a long phrase. The -1

above and to the right of the abbreviation of the name of the function is *not* an exponent, although it occupies the position of one. It may be used in connection with any one of the trigonometric functions.

APPLICATIONS OF TRIGONOMETRY

USE OF TRIGONOMETRIC TABLE

24. Finding Functions of an Angle.—The values of the various functions of angles are never calculated directly, except for the purpose of making a table of such values. For calculations that require the use of trigonometric functions, a table such as that given at the end of this Section is employed and the values of the functions are taken directly from it. The method of finding the natural sine, cosine, tangent and cotangent of an angle from the table will be illustrated by a number of examples and their solutions.

EXAMPLE 1.—Find the sine of an angle of 37° 24′.

SOLUTION.—The sines and cosines of angles are given in the first half of the table. Therefore, as 37° 24′ is less than 45°, look along the tops of the pages in the first half of the table until the value 37° is discovered, on page 8. Under this value are two columns, one headed Sine and the other Cosine. As the sine of the angle is to be found, its value will be in the column headed Sine. When the whole number of degrees in the angle is given at the *top* of the page, the minutes will be found in the first column at the left-hand edge of the page. Therefore, run down this column (which is headed ′) until the number 24 is located. Then follow horizontally along the row of figures to the column headed Sine under the value 37° already located. Here the value .60738 appears. Therefore, the sine of 37° 24′ is .60738. Ans.

EXAMPLE 2.—What is the cosine of 18° 52′?

SOLUTION.—Values of the cosine are found in the first half of the table. As the given angle is less than 45°, look along the tops of the pages until 18° is found. Run down the left-hand column until 52 is located and then proceed to the right, to the column headed Cosine under the value 18°, where the number .94627 appears. Then, the cosine of 18° 52′ is .94627. Ans.

EXAMPLE 3.—What is the value of tan 41° 8′?

SOLUTION.—The angle whose tangent is to be found is less than 45°. Therefore, run along the tops of the pages in the latter half of the

table until the value 41° is located. Follow down the left-hand column to 8 and then horizontally to the right to the column headed Tang under 41°, where .87338 is found. Then, tan 41° 8′=.87338. Ans.

EXAMPLE 4.—Find the value of cot 29° 40′.

SOLUTION.—The values of cotangents are given in the latter half of the table. As the angle is less than 45°, run along the tops of the pages until 29° is discovered. Then proceed downwards in the left-hand column to 40, from which follow horizontally to the right to the column headed Cotang under 29°, where the value 1.75556 appears. Then, cot 29° 40′=1.75556. Ans.

EXAMPLE 5.—What is the sine of 77° 43′?

SOLUTION.—The values of sines are found in the first half of the table. As the given angle is *greater* than 45°, it is necessary to look along the bottoms of the pages until the value 77° is discovered. When the whole number of degrees in the angle is found at the bottom, the number of minutes must be located in the column at the *right-hand* edge of the page. In this column, therefore, locate 43 and then proceed to the *left* to the column marked Sine at the *bottom* of the page and having 77° under it. Here the value .97711 appears. Then, sin 77° 43′ =.97711. Ans.

EXAMPLE 6.—Find the value of cos 82° 3′.

SOLUTION.—The first half of the table must be used, and as the given angle is greater than 45° it is necessary to follow along the bottoms of the pages until 82° is located. Proceed upwards in the column at the right-hand edge of the page to the value 3 and then to the left horizontally to the column marked Cosine at the *bottom* of the page and having 82° under it. Here the value .13831 will be found. Therefore, cos 82° 3′=.13831. Ans.

EXAMPLE 7.—Find the tangent of 50° 21′.

SOLUTION.—Tangents are found in the latter half of the table. As the given angle is greater than 45°, follow along the bottoms of the pages until the value 50° is discovered and then upwards along the right-hand column until 21 is located. Then, from 21, proceed to the left to the column marked Tang at the bottom and having 50° under it. Here the value 1.20665 will be found. Hence, tan 50° 21′=1.20665. Ans.

EXAMPLE 8.—Find the cotangent of 66° 6′.

SOLUTION.—The latter half of the table must be used. Follow along the bottoms of the pages until 66° is discovered. Next, proceed upwards along the right-hand column to 6 and then horizontally to the left to the column marked Cotang at the bottom and having 66° under it. Here the value .44314 is found. Therefore, cot 66° 6′=.44314. Ans.

EXAMPLE 9.—What is the sine of 30°?

SOLUTION.—The given angle contains a whole number of degrees and no minutes; hence, it may be written 30° 0′. As this is less than 45°, look along the tops of the pages in the first half of the table until 30° is discovered. Then, in the line having 0 at the left-hand end and in the column headed Sine, under 30°, is found the value of the sine, or .50000. Ans.

EXAMPLE 10.—Find the sine and cosine of the complement of 21° 38′.

SOLUTION.—According to Art. **17,** the sine of an angle is equal to the cosine of its complement and the cosine is equal to the sine of its complement. The complement of 21° 38′ is 90°—21° 38′; therefore, sin (90°—21° 38′)=cos 21° 38′=.92956, according to the table; and cos (90°—21° 38′)=sin 21° 38′=.36867. Ans.

EXAMPLE 11.—What are the cosine and tangent of 148° 50′?

SOLUTION.—As this angle is greater than 90° and less than 180°, proceed according to the last statement in Art. **18.** First subtract 148° 50′ from 180°, and the remainder is 31° 10′. Then, numerically, cos 148° 50′=cos 31° 10′=.85567, and tan 148° 50′=tan 31° 10′ =.60483. Ans.

EXAMPLE 12.—Find the sine of 17° 31′ 56″.

SOLUTION.—According to Art. **21,** this angle is taken as 17° 32′, approximately, as 56″ is greater than 30″. Then, the sine of 17° 32′, from the table, is .30126. Ans.

EXAMPLE 13.—What is the tangent of 59° 42′ 11″?

SOLUTION.—According to Art. **21,** the 11″ is ignored, as it is less than 30″, and the angle is taken as 59° 42′, approximately. From the table, tan 59° 42′=1.71129. Ans.

EXAMPLE 14.—What is the cosine of 37′ 37″?

SOLUTION.—As 37″ is more than 30″, the number of minutes is increased by 1, giving 38′, according to Art. **21.** Then, the value of the cosine of 38′ is found, in the column headed Cosine, under 0°, opposite 38 in the left-hand column, to be .99994. Ans.

EXAMPLE 15.—Find the sine and the cosine of 90°.

SOLUTION.—The table does not show 90° directly; but, as explained in Art. **20,** 90°=89° 60′, and the sine and the cosine of 89° 60′ may easily be found from the table. As 90°, or 89° 60′, is greater than 45°, look along the bottoms of the pages until 89° is found, on page 1. Follow upwards in the right-hand column to 60, which is in the top line of the table. In the two columns above 89°, marked Sine and Cosine at the bottom, and in the top line with 60, will be found the values 1 and

.00000. Therefore, sin 90°=sin 89° 60′=1, and cos 90°=cos 89° 60′ =.00000. Ans.

EXAMPLE 16.—What are the values of tan 90° and cot 90°?

SOLUTION.—As just explained, 90°=89° 60′. To find the tangent and cotangent of 89° 60′, first locate 89° at the bottom of page 10 in the table. Then follow upwards in the column at the right to 60, which is in the top line. In this line, and in the columns marked Tang and Cotang at the bottom, above 89°, are the values Infin. and .00000. Therefore, tan 90°=tan 89° 60′=infinity, and cot 90°=cot 89° 60′ =.00000. Ans.

The method of using the table, as explained in the foregoing solutions, may be stated in the form of rules, as follows:

Rule I.—I. *If the given angle is less than 45°, locate the figures that represent the number of degrees in the angle, as found at the top of the page, and below them find the column headed by the name of the function to be found.*

II. *Look down the column at the left-hand edge of the page until the figures representing the number of minutes in the given angle are found.*

III. *In the column headed by the name of the function, and on the same horizontal line as the number of minutes in the angle, note the number given, which is the value of the function to be found.*

Rule II.—I. *If the given angle is 45° or more, locate the figures that represent the number of degrees in the angle, as found at the bottom of the page, and above them find the column designated by the name of the function to be found.*

II. *Look up the column at the right-hand edge of the page until the figures representing the number of minutes in the given angle are found.*

III. *In the column designated at the bottom by the name of the function, and on the same horizontal line as the number of minutes in the angle, note the number given, which is the value of the function to be found.*

25. Finding Angles From Given Functions.—In many calculations the values of the functions of angles are

known and it is necessary to find the angles corresponding to those values of the functions. The table is used for this purpose, but the method of procedure is directly the reverse of that just described for finding the values of the functions. The table may not, and probably will not, show the exact value of the given function in every case; but it will be sufficiently accurate to locate the value nearest to that given and then find its corresponding angle.

First note carefully the arrangement of the table, for then it will be comparatively easy to find functions in it and the angles corresponding to them. At the tops of the columns appear numbers of degrees ranging from 0° to 4° on page 1 and so on up to 44° on page 9. When these values are used, the left-hand column is the one in which to find the number of minutes. Notice that the values of the sines increase from 0 for 0° 0′ to .01745 for 0° 60′ (or 1°), page 1, and so on up to .70711 for 44° 60′ (or 45°), page 9. The values of the cosines, on the other hand, decrease from 1 for 0° 0′ to .99985 for 0° 60′ (or 1°), page 1, and so on down to .70711 for 44° 60′ (or 45°), page 9.

The table is so arranged that each number on pages 1 to 9 represents the sine of an angle and the cosine of the complement of that angle. If the angle is larger than 45° the name of the function is given at the bottom of the column and the number of minutes in the right-hand column. Notice that the sines of angles greater than 45° continue to increase, as read upwards from .70711 for 45°, page 9, to .71934 for 45° 60′ (or 46°), and so on up to 1 for 89° 60′ (or 90°), page 1. Similarly read, the cosines decrease from .70711 for 45° to 0 for 89° 60′ (or 90°).

The tangents and cotangents are arranged in like manner on pages 10 to 18. The values of tangents increase from 0 for 0° 0′, page 10, to 1 for 44° 60′ (or 45°), page 18, and further increase, reading upwards from the bottoms of columns, to infinity for 89° 60′ (or 90°), on page 10. The values of cotangents vary in the reverse order from infinity for 0 down to 1 for 45° and, reading upwards and backwards from page 18, on down to 0 for 90°.

EXAMPLE 1.—Find the angle whose sine is .57691.

SOLUTION.—Values of sines are given in the first half of the table. Therefore, search through the columns headed Sine until the value .57691 is discovered. It will be found in the column bearing 35° at the top and in the line having 14 at the left-hand end. Therefore, the angle whose sine is .57691 is 35° 14′. Ans.

It will be observed that when the given function is found in a column having the name of that function at the *top*, then the number of degrees in the corresponding angle is the number *above* the name of the function, and the number of minutes is the number in the *left-hand* column in the same line as the function. When the given function is found in a column having the name of that function at the *bottom*, then the number of degrees in the corresponding angle is the number *below* the name of the function, and the number of minutes is the number in the *right-hand* column in the same line as the function.

EXAMPLE 2.—What angle has a sine whose value is .79300?

SOLUTION.—Look through the columns having Sine at the *top*. The greatest value discovered is .70711, which is at the end of the first half of the table and corresponds to an angle of 45°. As the given value, or .79300, is greater than .70711, it is plain that the angle must be greater than 45°, because the sine increases as the angle increases. Therefore, it is necessary to continue the search in the columns having Sine at the *bottom*. Following this plan, the value .79300 is discovered in the Sine column having 52° under it at the bottom of the page and on the line having 28 at the right-hand end. The number of minutes is found in the right-hand column because the given function appears in a column having the name of the function at the bottom. Therefore, the angle having a sine of .79300 is 52° 28′. Ans.

EXAMPLE 3.—The cosine of an angle is .36120. What is the angle?

SOLUTION.—Look through the columns having Cosine at the *top*. The values steadily decrease from 1 to .70711, the latter value being the cosine of 45°. As .70711 is still much greater than .36120, it is plain that the angle must be greater than 45°, because the value of the cosine decreases as the angle grows larger. Therefore, it is necessary to search in the columns having Cosine at the *bottom* of the page. Following this plan, two values are discovered that are close to .36120. These are .36135 and .36108. The first one is .00015 greater than .36120 and the second one is .00012 smaller than .36120. Thus, the second is closer to the given value, .36120, and is the one to be used. It is in the column having Cosine at the bottom and 68° below it, and it is on the

line having 50 in the right-hand column. Therefore, the required angle is 68° 50′, very closely. Ans.

EXAMPLE 4.—Find the angle whose tangent is .77777.

SOLUTION.—The exact value given does not appear in the table; but in one of the columns headed Tang at the *top* appear the values .77754 and .77801. The first of these values is the closer to .77777 and it is therefore used. It has the value 37° at the top of its column, and at the left-hand end of the line in which it occurs the value 52 appears. Therefore, the angle whose tangent is .77777 is 37° 52′ approximately. Ans.

EXAMPLE 5.—The cotangent of an angle is .41144. What is the angle?

SOLUTION.—The cotangent decreases as the angle increases. On looking through the columns having Cotang at the top, the smallest value found is 1.00000, corresponding to 45°. Therefore, the angle must be greater than 45°. Look through the columns having Cotang at the *bottom,* therefore, and eventually the value .41149 will be discovered, which is the nearest value to .41144 given in the table. This value .41149 corresponds to an angle of 67° 38′. Therefore, the required angle is 67° 38′, very nearly. Ans.

EXAMPLES FOR PRACTICE

1. Find the sine of 48° 17′. Ans. .74644
2. Find the cosine of 13° 11′. Ans. .97365
3. What is the sine of 72°? Ans. .95106
4. Find the value of cos 51° 9′. Ans. .62728
5. What is the tangent of 30°? Ans. .57735
6. Find the cotangent of 82° 50′. Ans. .12574
7. (*a*) Of what angle is .26489 the sine? (*b*) Of what angle is it the cosine?
   Ans. (*a*) 15° 22′; (*b*) 74° 38′
8. (*a*) Of what angle is .68814 the sine? (*b*) Of what angle is it the tangent?
   Ans. (*a*) 43° 29′; (*b*) 34° 32′
9. The cosine of an angle is .36257. What is the angle?
   Ans. 68° 45′
10. The tangent of an angle is 1.43916. Find the angle.
    Ans. 55° 12′
11. Find the angle whose cotangent is 2.12200. Ans. 25° 14′
12. What is the complement of 39° 42′? Ans. 50° 18′

13. What is the supplement of 117° 22′? Ans. 62° 38′
14. Find the sine of 153° 45′. Ans. .44229
15. What is the cosine of the complement of 61° 30′? Ans. .87882
16. What is the tangent of 12° 48′ 10″? Ans. .22719
17. Find the sine of 74° 36′ 58″ Ans. .96417
18. What is the cosine of 53′ 42″ Ans. .99988
19. What is the sine of 28′ 12″? Ans. .00814
20. Find the cotangent of 65° 7′ 40″. Ans. .46348

SOLUTION OF RIGHT TRIANGLES

26. Cases Involved.—It has already been stated that if three parts of a triangle are known, one of them being a side, the remaining three parts may be calculated. To illustrate this calculation, known as the solution of the triangle, problems relating to right triangles will first be considered. Every right triangle has one angle of 90°; consequently, the sum of the two remaining acute angles must be 90°, because the sum of all three angles is 180°. Moreover, the sum of the squares of the two sides is equal to the square of the hypotenuse. These two statements should be kept in mind, as they assist materially in solving right angles.

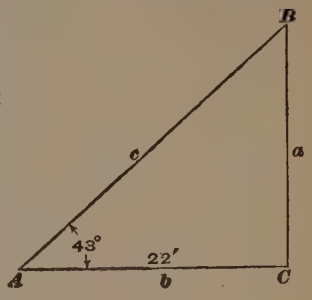

FIG. 5

There are four separate cases that may arise in the solution of right triangles, and they depend on what parts of the triangle are known. These cases are:

1. An acute angle and one side given.
2. An acute angle and the hypotenuse given.
3. The hypotenuse and one side given.
4. The two sides given.

In each of these cases it will be noted that only two parts are given; but, as every right triangle has one right angle, this

right angle forms the third known part and so enables the triangle to be solved.

27. Given an Acute Angle and a Side.—In the right triangle shown in Fig. 5, the acute angle A is 43° and the side b is 22 feet long. Let it be required to find the other parts. As the sum of the two acute angles must be 90°, the angle B must equal $90° - A = 90° - 43° = 47°$. According to trigonometry, $\frac{a}{b} = \tan A = \tan 43°$. But, $b = 22$ feet, and $\tan 43° = .93252$, according to the table. Hence, $\frac{a}{22} = .93252$, or $a = 22 \times .93252 = 20.5$ feet. The square of the hypotenuse equals the sum of the squares of the two sides, or, $c^2 = a^2 + b^2 = 22^2 + 20.5^2 = 484 + 420.25 = 904.25$. Then, $c = \sqrt{904.25} = 30.07$ feet, or the length of the hypotenuse.

If the angle A and the side a had been given, the angle B would have been calculated as before. Then, as $\frac{b}{a} = \tan B$, the value of b could be found in precisely the same manner as that just described.

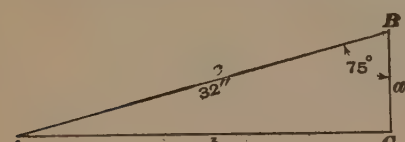

FIG. 6

28. Given an Acute Angle and the Hypotenuse.—Suppose that the hypotenuse c, Fig. 6, is 32 inches long and the angle B is 75° and that the other parts of the triangle are to be found. The angle $A = 90° - B = 90° - 75° = 15°$. Then, $\frac{a}{c} = \sin A = \sin 15°$. But, $c = 32$ inches and $\sin 15° = .25882$, according to the table. Hence, $\frac{a}{32} = .25882$, or $a = 32 \times .25882 = 8.3$ inches. Similarly, $\frac{b}{c} = \sin B = \sin 75°$; or, $\frac{b}{32} = .96593$, from which $b = 32 \times .96593 = 30.9$ inches. Another way of finding b is as follows: $c^2 = a^2 + b^2$, or $b^2 = c^2 - a^2$. Then, $b^2 = 32^2 - 8.3^2 = 1{,}024 - 68.9 = 955.1$, and $b = \sqrt{955.1} = 30.9$ inches, as before.

If the acute angle A had been given instead of B, the angle B would have been found by subtracting A from 90°, after which the remaining parts would have been calculated precisely as shown.

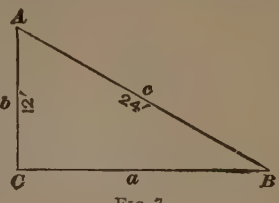

Fig. 7

29. Given the Hypotenuse and One Side.—Suppose that the triangle in Fig. 7 is to be solved, knowing that $b=12$ feet and $c=24$ feet. The side a is readily found as follows: $a^2=c^2-b^2=24^2-12^2=576-144=432$, and $a=\sqrt{432}=20.78$ feet. Then, $\sin A=\frac{a}{c}=\frac{20.78}{24}=.866$. From the table, the angle whose sine is .866 is found to be 60°; therefore, $A=60°$. Then, $B=90°-A=90°-60°=30°$.

If the side a and the hypotenuse c had been given, the side b would have been found by the formula $b=\sqrt{c^2-a^2}$ and the remainder of the solution would have been as shown.

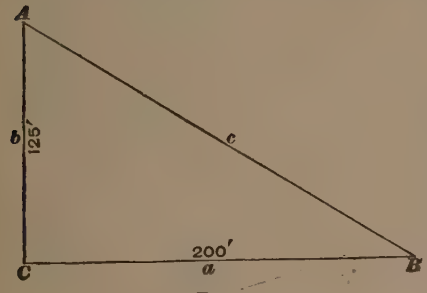

Fig. 8

30. Given the Two Sides.—In Fig. 8, a is 200 feet long and b is 125 feet long and the hypotenuse and the angles are to be found. First, $c^2=a^2+b^2=200^2+125^2=40,000+15,625=55,625$, and so $c=\sqrt{55,625}=235.8$ feet. As a and b are known, $\tan A=\cot B=\frac{a}{b}=\frac{200}{125}=1.6$. The table shows that 1.6 is the tangent of 58° and the cotangent of 32°; therefore, $A=58°$ and $B=32°$.

EXAMPLES FOR PRACTICE

1. In a right triangle $A\,B\,C$, having the right angle at C, the hypotenuse c is 24 feet and the angle A is 29° 31′. Find the sides a and b and the angle B.

Ans $\begin{cases} a=11.8 \text{ ft.} \\ b=20.9 \text{ ft.} \\ B=60° 29' \end{cases}$

NOTE.—When solving an example of this kind, make a rough sketch of a right triangle and letter the angles and sides as stated in the example.

2. The sides a and b of a right triangle have lengths of 15 feet and 18 feet, respectively. Find the length of the hypotenuse c and the values of the acute angles A and B.

Ans. $\begin{cases} c=23.4 \text{ ft.} \\ A=39° 48' \\ B=50° 12' \end{cases}$

3. In a right triangle $A\,B\,C$, having the right angle at C, the side a is 37 feet 7 inches long and the angle A is 25° 33′. Find the angle B and the lengths of the side b and hypotenuse c.

Ans. $\begin{cases} B=64° 27' \\ b=78.6 \text{ ft.} \\ c=87.1 \text{ ft.} \end{cases}$

4. The hypotenuse c of a right triangle measures 596.76 feet and the side b measures 305.45 feet. Find the length of the side a and the angles A and B.

Ans. $\begin{cases} a=512.7 \text{ ft.} \\ A=59° 13' \\ B=30° 47' \end{cases}$

SOLUTION OF OBLIQUE TRIANGLES

31. Cases Involved.—Any triangle that is not a right-angled triangle is called an **oblique triangle.** In the solution of such a triangle, four cases may arise, as follows:

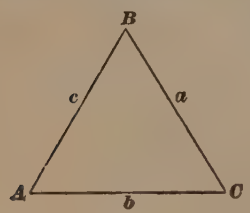

FIG. 9

1. A side and two angles given.
2. Two sides and an angle opposite one of them given.
3. Two sides and the angle between them given.
4. The three sides given.

32. Law of Sines.—The solution of oblique triangles is greatly simplified by using the law of sines. This law states that *the ratio between any two sides of a triangle equals the ratio between the sines of the angles opposite those sides.* If

the triangle in Fig. 9 is considered, this law means that $\frac{a}{b}=\frac{\sin A}{\sin B}$, $\frac{b}{c}=\frac{\sin B}{\sin C}$, $\frac{c}{a}=\frac{\sin C}{\sin A}$, etc. But, if $\frac{a}{b}=\frac{\sin A}{\sin B}$, $a\times\sin B=b\times\sin A$, and $\frac{a}{\sin A}=\frac{b}{\sin B}$. Therefore, another way in which the law may be stated is: *The sides of any triangle are proportional to the sines of the opposite angles.* In other words, the ratios obtained by dividing each of the sides by the sine of the angle opposite will be equal; that is, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

The sine law is very important and useful.

33. Given a Side and Two Angles.—Suppose that the angles A and B, Fig. 10, are 23° 17′ and 74° 49′, respectively, and the side a has a length of 182 feet, and let it be required to find the remaining parts of the triangle. As the sum of all three angles must be 180°, the angle $C=180°-(A+B)=180°-(23°\ 17'+74°\ 49')=180°-98°\ 6'=81°\ 54'$. By the law of sines, $a:\sin A=b:\sin B$; but, $a=182$ feet, $\sin A=\sin 23°\ 17'=.39528$, and $\sin B=\sin 74°\ 49'=.96509$. Then, $182:.39528=b:.96509$, or $b=\frac{182\times.96509}{.39528}=444.3$ feet. In the same way, $a:\sin A=c:\sin C$, or $a:\sin 23°\ 17'=c:\sin 81°\ 54'$, from which $182:.39528=c:.99002$; then, $c=\frac{182\times.99002}{.39528}=455.8$ feet.

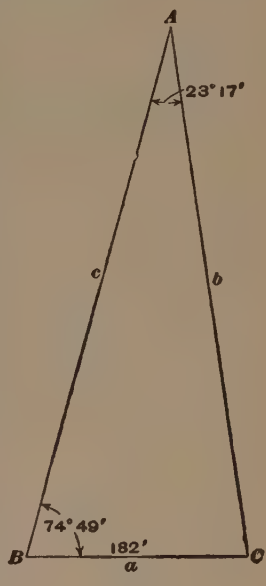

Fig. 10

34. Given Two Sides and an Opposite Angle.—In the triangle in Fig. 11, the sides a and b measure 115 feet and 164 feet, respectively, the angle B is 28° 40′, and the other parts are to be found. By the law of sines, $a:\sin A=b:\sin B$, or

$115 : \sin A = 164 : \sin 28° 40'$, from which $\sin A = \dfrac{115 \times \sin 28° 40'}{164}$ $= \dfrac{115 \times .47971}{164} = .33638$. The angle whose sine is .33638 is approximately 19° 39′, which is therefore the value of the angle A. The angle C is then $180° - (19° 39' + 28° 40') = 131° 41'$. By the law of sines, $b : \sin B = c : \sin C$. The sine of 131° 41′ is the same as the sine of $(180° - 131° 41' = 48° 19')$, according to Art. **18**, and the sine of 48° 19′ is .74683. Then, $164 : .47971 = c : .74683$, or $c = \dfrac{164 \times .74683}{.47971} = 255.3$ feet.

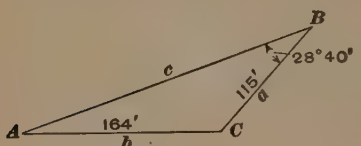

Fig. 11

35. In the example stated in the preceding article there is but one triangle that can be formed, having the given sides and angle. But in some cases two triangles can be made to fit the given conditions. For example, suppose that the given sides are $a = 20$ inches and $c = 24$ inches, and that the angle $A = 50° 30'$. In Fig. 12 (a) is shown one triangle that will fulfil the

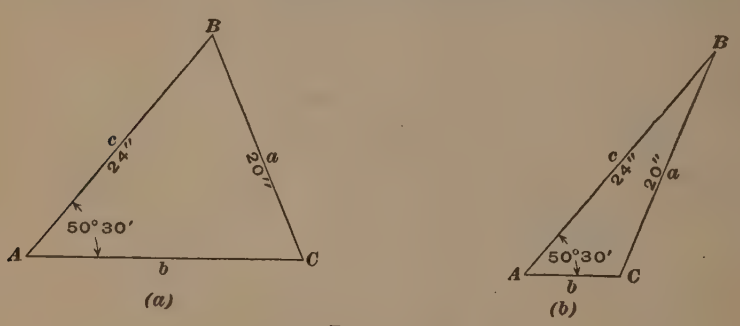

Fig. 12

conditions and in (b) is shown another. In each of them, the angle A is 50° 30′ and the sides a and c measure 20 and 24 inches. But in the first case the angle C is an acute angle and in the second case it is obtuse; also, the angle B varies in the two triangles. Therefore, two sets of values must be obtained for the side b and the angles B and C.

By the law of sines, a : sin $A = c$: sin C, or 20 : sin 50° 30′ = 24 : sin C, from which sin $C = \frac{24 \times .77162}{20} = .92594$. Therefore, the angle C is 67° 49′ if it is an acute angle, as in (a). But, the sine of an angle is also equal to the sine of 180° minus that angle, according to Art. **18.** Hence, .92594 is also the sine of an angle of $(180° - 67° 49′ = 112° 11′)$, which is the value of the obtuse angle C in the triangle shown in (b).

Consider first the triangle in (a), in which the angle C is 67° 49′. The angle B is equal to 180° minus the sum of the angles A and C; that is, the angle $B = 180° - (50° 30′ + 67° 49′) = 61° 41′$. According to the law of sines, b : sin $B = a$: sin A, or b : sin 61° 41′ = 20 : sin 50° 30′, from which $b = \frac{20 \times .88034}{.77162} = 22.8$ inches.

Next consider the triangle in (b), in which $C = 112° 11′$. The angle $B = 180° - (50° 30′ + 112° 11′) = 17° 19′$. Then, by the law of sines, b : sin $B = a$: sin A, or b : sin 17° 19′ = 20 : sin 50° 30′, from which $b = \frac{20 \times .29765}{.77162} = 7.7$ inches.

36. Given Two Sides and the Included Angle. The angle A, Fig. 13, is 57° 32′ and the sides b and c on each

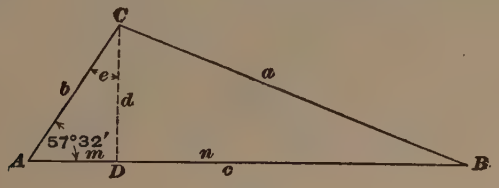

Fig. 13

side of it measure 12.5 inches and 33.25 inches, respectively. It will be seen at once that no side and opposite angle together are known and so it is not possible to use the law of sines directly, since the required ratio cannot be found. However, the triangle may be solved as follows: From C draw $C\,D$ perpendicular to $A\,B$, thus dividing the oblique triangle into two right triangles $A\,C\,D$ and $B\,D\,C$. Denote the distance $C\,D$ by d and the two parts into which the side c is divided by m and n.

Considering, then, the right triangle $A\,C\,D$, the angle $e = 90° - 57°\ 32' = 32°\ 28'$. Also $\frac{m}{b} = \sin e$, or $\frac{m}{12.5} = \sin 32°\ 28' = .53681$ and so $m = 12.5 \times .53681 = 6.7$ inches. Similarly $\frac{d}{b} = \sin 57°\ 32'$, or $\frac{d}{12.5} = .84370$, from which $d = 12.5 \times .84370 = 10.55$ inches.

Next, consider the right triangle $B\,D\,C$. The side d has just been found to be 10.55 inches, and the side $n = c - m = 33.25 - 6.7 = 26.55$ inches. Consequently, as a is the hypotenuse, $a = \sqrt{d^2 + n^2} = \sqrt{10.55^2 + 26.55^2} = 28.6$ inches, nearly.

Now consider the triangle $A\,B\,C$. By the law of sines, $a : \sin A = b : \sin B$; but $a = 28.6$ inches, $b = 12.5$ inches, and $\sin A = \sin 57°\ 32' = .84370$. Therefore, $28.6 : .84370 = 12.5 : \sin B$, from which $\sin B = \frac{12.5 \times .84370}{28.6} = .36875$. As .36875 is the sine of an angle of 21° 38′, nearly, $B = 21°\ 38'$. Finally, then, the angle $C = 180° - (57°\ 32' + 21°\ 38') = 100°\ 50'$.

37. Given the Three Sides.—If the lengths of the three sides of a triangle are given, but no angles are known, it is not possible to use the law of sines in the solution, because no ratio can be formed between a side and the sine of the angle opposite. However, there is a **law of cosines** that enables the triangle to be solved very readily. This law states that *the cosine of any angle of a triangle is equal to the sum of the squares of the sides adjacent to that angle minus the square of the side opposite, all divided by twice the product of the sides adjacent to the angle.* Thus, if A, B, and C are the three angles of a triangle and a, b, and c are the sides opposite these angles, respectively, the law of cosines may be written for each of the three angles, as follows:

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c} \qquad (1)$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2\,a\,c} \qquad (2)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2\,a\,b} \qquad (3)$$

These formulas are important and are not difficult to remember. Their application will be illustrated by an example.

38. Suppose that the three angles of the triangle in Fig. 14 are to be found, if the sides a, b, and c have lengths of

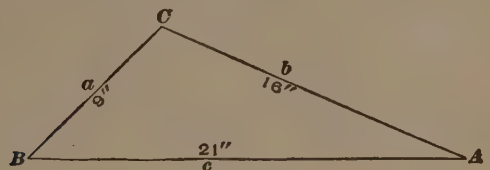

FIG. 14

9, 16, and 21 inches, respectively. First consider the angle A; apply formula **1** of the preceding article and substitute the values of a, b, and c in that formula. Then, $\cos A = \frac{b^2+c^2-a^2}{2bc}$ $= \frac{16^2+21^2-9^2}{2\times16\times21} = \frac{256+441-81}{672} = \frac{616}{672} = .91667$. This value, according to the table, is the cosine of an angle of 23° 33′; therefore, $A = 23°\ 33'$. Next, consider the angle B and apply formula **2** of the preceding article. Then, $\cos B = \frac{a^2+c^2-b^2}{2ac}$ $= \frac{9^2+21^2-16^2}{2\times9\times21} = \frac{266}{378} = .70370$, which is the cosine of an angle of 45° 17′; therefore, $B = 45°\ 17'$. The angle C can be found by applying formula **3** of the preceding article; but as the angles A and B are known, it is simpler to find C by subtracting the sum of A and B from 180°. Thus, $C = 180° - (23°\ 33' + 45°\ 17') = 180° - 68°\ 50' = 111°\ 10'$.

EXAMPLES FOR PRACTICE

1. In a triangle ABC, the angle A is 46° 14′, the angle B is 88° 24′ and the side c is 21 inches. Find the angle C and the sides a and b.

Ans. $\begin{cases} C = 45°\ 22' \\ a = 21.3 \text{ in.} \\ b = 29.5 \text{ in.} \end{cases}$

NOTE.—In solving an example of this kind, make a sketch of a triangle and mark the sides and angles as indicated by the example.

2. In a triangle $A\,B\,C$, the side c is 18 inches, the angle B is 60°, and the angle C is 38° 42′. Find the angle A and the sides a and b.

Ans. $\begin{cases} A=81°\ 18' \\ a=28.5 \text{ in.} \\ b=24.9 \text{ in.} \end{cases}$

3. In a triangle $A\,B\,C$, the angle A is 35° 1′, the side a is 57 feet, and the side c is 88½ feet. Find the side b and the angles B and C.

Ans. $\begin{cases} b=46.6 \text{ ft.} \\ B=27°\ 58' \\ C=117°\ 1' \end{cases}$ or $\begin{cases} b=98.4 \text{ ft.} \\ B=82° \\ C=62°\ 59' \end{cases}$

NOTE.—Two triangles may be found to fulfil the conditions of this example. In one the angle C is acute, and in the other it is obtuse. Follow the method in Art. **35.**

4. If the three sides of a triangle are $a=17.25$ feet, $b=21$ feet, and $c=32$ feet, find the angles A, B, and C.

Ans. $\begin{cases} A=29°\ 42' \\ B=37°\ 6' \\ C=113°\ 12' \end{cases}$

TRIGONOMETRIC CALCULATIONS

39. The use of trigonometric functions in making certain calculations is most easily shown by illustrative examples such as the following:

EXAMPLE 1.—In the parallelogram shown in Fig. 15, the four sides are of equal lengths and the longer diagonal forms equal angles of 30° each with the sides. If a represents the length of the longer diagonal and b the length of one side, find the relation between a and b.

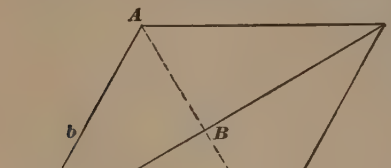

FIG. 15

SOLUTION.—Draw the other diagonal of the parallelogram, as shown dotted. It will cut the longer diagonal at its middle point and the two diagonals will be at right angles. Therefore, the triangle $A\,B\,C$ will be a right triangle, in which the angle $A\,C\,B$ is 30°, the hypotenuse is b, and the adjacent side $C\,B$ is half the total length a of the longer diagonal, or $\frac{1}{2}\,a$. As the cosine of the angle $A\,C\,B$ is equal to the ratio between the side adjacent and the hypotenuse, and $A\,C\,B=30°$,

$$\cos A\,C\,B=\cos 30°=\tfrac{1}{2}a\div b=\tfrac{1}{2}\,\frac{a}{b}=\frac{a}{2\,b}$$

from which

$$a=2\,b\times\cos 30°$$

But, the cosine of 30° is .86603, according to the table; therefore,

$$a=2\,b\times.86603=1.73206\,b$$

or, expressed to three decimal places,

$$a=1.732\,b$$

which is the required relation of a to b. Ans.

Example 2.—A wooden brace a, Fig. 16, is to be set against a vertical post b to support a horizontal arm c. The distance from the post to the upper toe of the brace is to be 30 inches and from the arm to the lower toe of the brace is to be 48 inches. Find the angles m and n to which the ends of the brace must be sawed so as to fit.

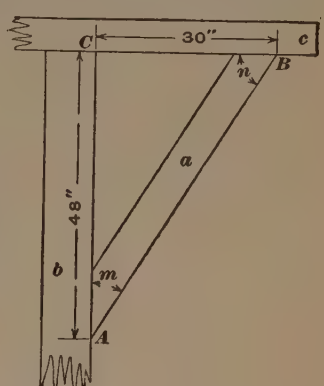

Fig. 16

Solution.—The triangle ABC formed by the brace and the arm and the post is a right triangle in which the lengths of the two sides are known and the acute angles m and n are to be found. Consider first the angle m; its tangent is equal to the side opposite divided by the side adjacent; that is,

$$\tan m=\frac{30}{48}=.625$$

In the table of tangents and cotangents, the angle whose tangent is .625 is found to be 32°; therefore, the lower end of the brace must be cut to an angle of 32°.

As the sum of the acute angles of a right triangle is always 90°, the angle n has a value of 90°—32° =58°. Ans.

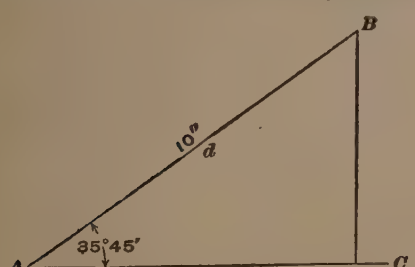

Fig. 17

Example 3.—Two lines AB and AC, Fig. 17, make an angle A of 35° 45′ with each other. If a line is drawn from B perpendicular to AC at D, and AB is 10 inches long, find the length of AD.

Solution.—The triangle ABD is a right triangle, with a right angle at D. Let b represent the length of AD and d the length of AB, or 10 in. Then, $\frac{b}{d}=\cos A$, or $\frac{b}{10}=\cos 35°\ 45'=.81157$, from which $b=10\times.81157=8.1157$, or 8.12 in., approximately. Ans.

EXAMPLE 4.—A man standing on the bank of a river, as at A, Fig. 18, is directly opposite the downstream end B of a pier that is being built for a bridge. The line of sight $A\ C$ to the upstream end C

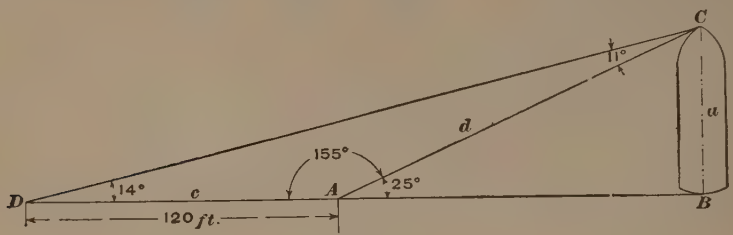

FIG. 18

of the pier makes an angle of 25° with $A\ B$. He moves back in a direct line 120 feet to a point D and then the line of sight $D\ C$ makes an angle of 14° with $D\ B$. What is the length $B\ C$ of the pier?

SOLUTION.—The angle $C\ A\ B$=25°; but, the angle $C\ A\ B$+the angle $C\ A\ D$=180°, or two right angles, because, according to geometry, the sum of all the angles at a point on one side of a straight line is 180°, or two right angles. Hence, the angle $C\ A\ D$=180°−25°=155°. In the oblique triangle $D\ A\ C$, therefore, the length of the side $D\ A$ is known to be 120 ft. and the angles at D and A are known to be 14° and 155°,

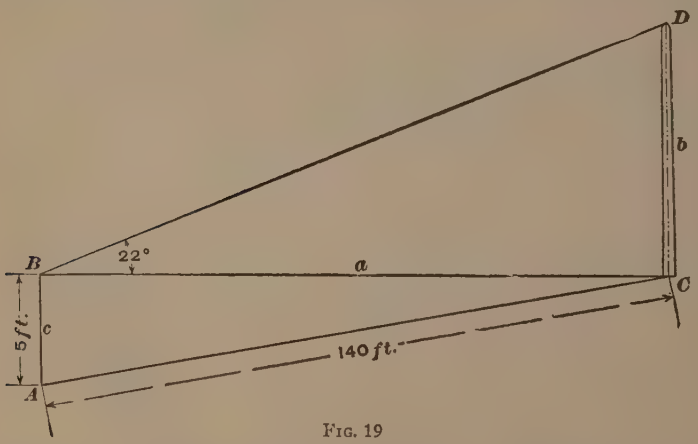

FIG. 19

respectively. As the sum of the three angles of a triangle is 180°, the angle $D\ C\ A$ must equal 180°−(155°+14°)=11°. According to the law of sines, d : sin 14°=c : sin 11°, or d : .24192=120 : .19081, from which $d=\frac{120\times.24192}{.19081}$=152 ft.

The triangle $A\,B\,C$ is a right triangle, in which the angle at A is 25° and the hypotenuse d=152 ft. Now, $\frac{a}{d}$=sin 25°, or $\frac{a}{152}$=.42262, from which a=152×.42262=64 ft. That is, the length of the pier is 64 ft. Ans.

Example 5.—When a man stands at a point A, Fig. 19, on a hillside, his eye is at B, on a level with the lower end C of a telephone pole on the hill above him, and the line of sight $B\,D$ to the top of the pole makes an angle of 22° with the horizontal $B\,C$. If the point at which the man stands is 140 feet from the base of the pole, and his eye is 5 feet above the same point, what is the height of the pole?

Solution.—The lines indicated in the illustration form two right triangles $A\,B\,C$ and $B\,C\,D$. Consider first the triangle $A\,B\,C$. The length of the hypotenuse is 140 ft. and of the side c is 5 ft. Therefore, the side $a=\sqrt{140^2-5^2}=\sqrt{19{,}600-25}=\sqrt{19{,}575}=139.9$ ft. Now consider the triangle $B\,C\,D$. The angle B=22° and the side a=139.9 ft. Consequently, $\frac{b}{a}$=tan B, or $\frac{b}{139.9}$=tan 22°=.40403, from which b=139.9 ×.40403=56.5 ft. Ans.

EXAMPLES FOR PRACTICE

1. A man standing near the base of a tree, with his eye on a level with its base, has to look up at an angle of 55° to see the top. If his eye is 28 feet from the tree, how high is the tree? Ans. 40 ft., nearly

Note.—The line from the eye to the base of the tree is the base of a right triangle, of which the line of sight to the top of the tree is the hypotenuse.

2. If the distances $C\,B$ and $C\,A$, Fig. 16, had been 28 inches and 40 inches, respectively, what would have been the values of the angles m and n? Ans. $\begin{cases} m=35° \\ n=55° \end{cases}$

3. Two of the sides of a triangle have lengths of 38 feet and 52 feet, respectively, and the angle opposite the latter side is 74° 19′. What is the length of the third side? Ans. 47 ft., approximately

4. A pole leans at an angle of 78° to the horizontal and a line hanging vertically from its top strikes the ground 9½ feet from the base. What is the length of the pole from the base to the top? Ans. 45.7 ft.

5. The shadow cast by a chimney is 168 feet long, measured from the base of the chimney, on level ground, and the height of the chimney is known to be 280 feet. Find the angle between the ground and a line from the top of the chimney to the tip of the shadow. Ans. 59° 2′

GRAPHS

METHOD OF CONSTRUCTION

INTRODUCTION

40. Definition.—A **graph** is a line that shows in diagrammatic, or graphic, form the relation between two quantities. It may be a straight line, a smooth curve, a broken line, or an irregular curved line, as will be illustrated later. Because of the fact that the greater number of graphs take the form of curved lines, they are commonly called **curves.**

41. Usefulness of Graphs.—Graphs are very useful in connection with engineering work; by means of them data can

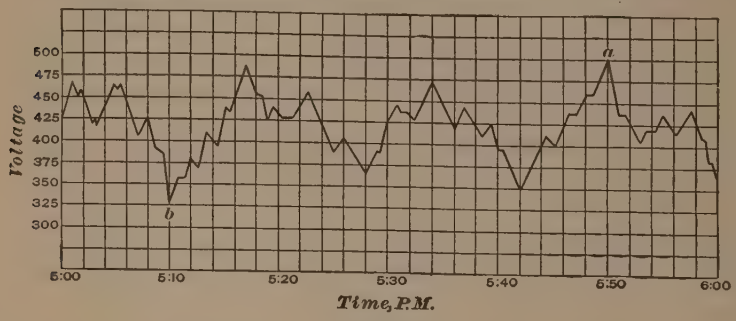

FIG. 20

be recorded in the form of diagrams that are not only easily and quickly understood but that oftentimes occupy much less space than would otherwise be required.

42. Simple Example of Graph.—A graph is shown in Fig. 20, in which the zigzag line extending from the left-hand edge of the diagram to the right-hand edge indicates the variations in the voltage in an electric line at succeeding periods of time. It will be observed that the line, or graph, rises and

falls. A rise in the line indicates an increase in the voltage and a drop indicates a decrease. The exact value of the voltage at any point along the graph may readily be found by means of the scale at the left-hand edge. Another scale along the bottom of the diagram indicates the time, the period indicated by the entire graph, or curve, being one hour, from 5 to 6 P. M. To show the value of the graph, suppose that it is desired to know when the voltage was greatest. A glance shows that the highest point of the graph is at *a*, which is on the vertical line that passes through *5:50;* hence, the voltage was greatest at 5:50. Again, suppose that the least value of the voltage is required, as well as the time at which it occurred. The lowest point of the graph is at once seen to be at *b*, which is on the horizontal line passing through *325* and on the vertical line passing through *5:10*. Therefore, the least value of the voltage was 325, which occurred at 5:10.

43. The usefulness of a graph of this kind may readily be explained. Suppose that, instead of a curve, the values of the voltage had been arranged in the form of a table, for every minute between 5:00 and 6:00. There would then have been 61 separate numbers, corresponding to the minute intervals of time. In order to discover the greatest voltage from the table, it would have been necessary to go over the entire list of 61 numbers, to discover which was the largest; whereas, by having a curve, or graph, the maximum value is quickly discovered by observing the highest point. The same saving of time is made in finding the least value of the voltage, by noting the lowest point of the curve.

44. Essentials of a Graphic Chart or Diagram. From a study of the features of the graph shown in the diagram, Fig. 20, several points are observed. In the first place, the meaning or value of any one point on the graph or curve depends on its position. The higher the point, the greater is the value of the voltage indicated by it; and the farther to the right it occurs, the later is the time indicated. In other words, the position and value of the point are fixed by its distance from each of two lines, one being the lowest horizontal line or

base of the diagram and the other being the left-hand vertical line at the edge of the diagram. These lines are drawn at right angles and other lines are then drawn parallel to them, dividing the diagram into a large number of small squares, to enable the positions of points on the curve to be determined quickly. These two lines, to which the position of each point must be referred, are called **reference lines, or axes of reference.**

45. A second point to be observed in connection with Fig. 20 is that the axes of reference are marked with certain values by which the position of any point is definitely determined. Along the horizontal axis of reference the several numbers indicate time, the numbers increasing in regular order by intervals of 10 minutes. Along the vertical axis of reference the numbers indicate voltage and these also are arranged in regularly increasing order. These two sets of numbers are the **scales** to which the curve or graph is drawn. The first one mentioned is the time scale and the other is the voltage scale. It will be seen that they are not alike. For example, the numbers on the voltage scale increase by 25, reading upwards; that is, each square indicates 25 volts. Along the time scale the difference between successive numbers is 10 minutes and the distance corresponds to five small squares; therefore, each small square indicates a value of $10 \div 5 = 2$ minutes.

PLOTTING OF POINTS

46. Coördinate Axes.—Before going into a study of various forms of graphs, it is necessary to have some understanding of how a graph is made. The first essential is a pair of axes of reference. If they are two straight lines of indefinite lengths, crossing at right angles, as $X\,O\,X'$ and $Y\,O\,Y'$, Fig. 21, they are called **coördinate axes** and each of the two lines is an axis. In order to make it easy to refer to each, $X\,O\,X'$ is known as the *X axis, axis of X,* or *horizontal axis,* and $Y\,O\,Y'$ is called the *Y axis, axis of Y,* or *vertical axis.* The point O at which they cross is called the **origin.**

47. Quadrants.—The crossing of two coördinate axes, as in Fig. 21, produces four right-angled openings, and these are known as **quadrants.** To enable them to be distinguished from one another they are numbered in the direction indicated by the arrow, beginning with the upper right-hand angle. Thus, $X\ O\ Y$ is called the first quadrant, $Y\ O\ X'$ the second quadrant, $X'\ O\ Y'$ the third quadrant, and $Y'\ O\ X$ the fourth quadrant.

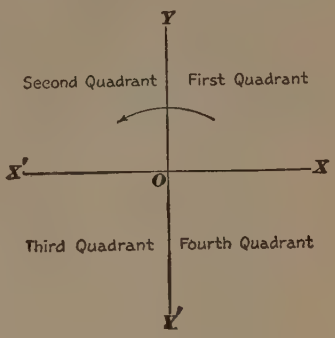

Fig. 21

48. Abscissas and Ordinates.—Suppose that a point, as P, Fig. 22, is placed at random in the first quadrant. If lines are drawn from P parallel to the two axes, a rectangle $P\ a\ O\ b$ will be formed, in which $P\ a$ will equal $O\ b$ and $P\ b$ will equal $O\ a$. The distance $O\ b$ $(=P\ a)$ that represents the distance of P from the Y axis is called the **abscissa** of the point; and the distance $P\ b$ that represents its distance from the X axis is called the **ordinate** of that point. The position of any point with respect to the coördinate axes can therefore be found at once, if its abscissa and ordinate are known. These distances are commonly designated by x and y, respectively. Suppose, for example, that for a certain point $x=1$ inch and $y=\frac{3}{4}$ inch, and that the position of the point is to be fixed.

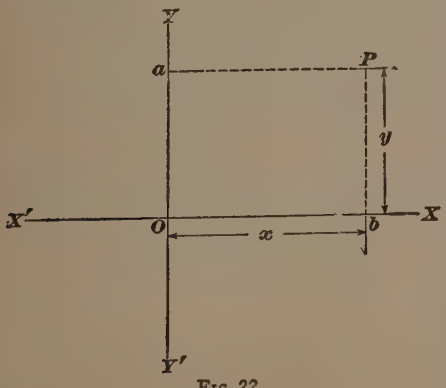

Fig. 22

As x is the abscissa, measure off a distance $O\ b$ equal to 1 inch, and above the point b thus located measure off $b\ P$ equal to $\frac{3}{4}$ inch, in a direction at right angles to $O\ b$, or parallel to the

Y axis. Then, the correct location of the point P is determined. Any point may be located in the same way, if the values of its x and y are given—that is, its abscissa and ordinate. The two values of the abscissa and ordinate are frequently referred to as the **coördinates** of the point. Thus, the coördinates of P are 1 inch and $\frac{3}{4}$ inch.

49. Signs in the Quadrants.—In the particular case indicated in Fig. 22, the point lies in the first quadrant, so that its abscissa is measured to the right of the Y axis and its ordinate above the X axis. But, as there are three other quadrants, in any one of which points may be located, it is necessary to have some way of marking the abscissas and ordinates so that the proper quadrant may be known at once. The commonly accepted way of doing this is to prefix a plus sign, $+$, or a minus sign, $-$, to the value of the abscissa, the ordinate, or both. The rule that has been established by custom is to consider as positive, or $+$, all measurements to the *right* of the Y axis or *above* the X axis, and to consider as negative, or $-$, all measurements to the *left* of the Y axis or *below* the X axis. Following this plan, the abscissas and ordinates in the several quadrants will have values either positive or negative, as follows:

Quadrant	Abscissa	Ordinate
First	Positive	Positive
Second	Negative	Positive
Third	Negative	Negative
Fourth	Positive	Negative

50. Designation of a Point.—The customary way of designating a point with reference to coördinate axes is to write down the values of the abscissa and ordinate, separated by a comma, the abscissa always being written *first,* and the proper signs being used to indicate whether the values are positive or negative. For example, the values $+4$, $+6$ indicate a point whose abscissa is $+4$ and whose ordinate is $+6$, and as both of these values are $+$, or positive, the point lies in the first quadrant, at a distance of 4 from the Y axis and at a distance of 6 from the X axis. To save time and space, the

plus sign is dropped when the value of the ordinate or the abscissa is positive; thus, 5, 1 indicates a point whose abscissa is $+5$ and whose ordinate is $+1$; -4, 3 indicates a point whose abscissa is -4 and whose ordinate is $+3$; and so on. In other words, if no sign is given before a number, the distance indicated is measured either above the X axis or to the right of the Y axis; and if a minus sign appears, the value is measured below the X axis or to the left of the Y axis.

51. Squared Paper.—The process of locating a point with reference to its coördinate axes is called **plotting.**

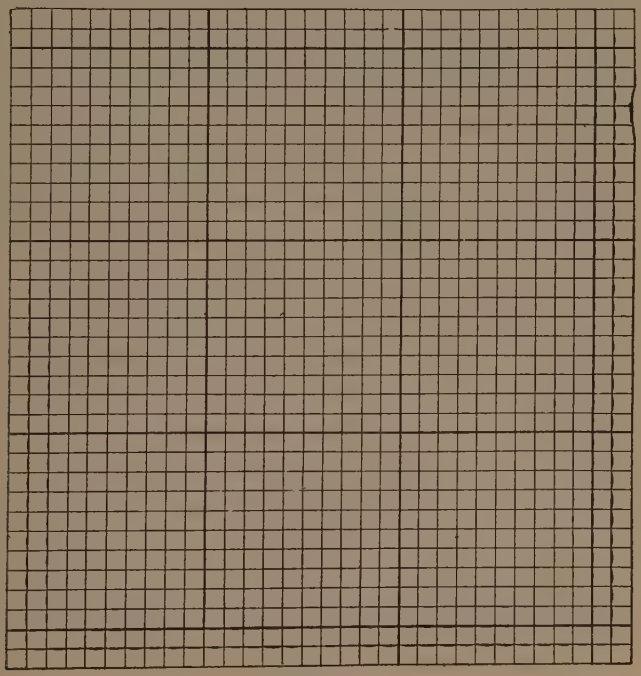

Fig. 23

Thus, the point 3, 7 may be plotted by measuring off 3 spaces or units to the right of the Y axis and 7 spaces or units above the X axis. The work of doing this, however, is made much more rapid and easy by the use of **squared paper,** known also as *cross-section paper* and *sectional paper.* It consists of sheets

of paper on which are ruled or printed lines at right angles, there being a variety of rulings to suit different purposes. One style commonly employed contains lines spaced $\frac{1}{10}$ inch apart, as shown in Fig. 23, the sheet being thus divided into squares measuring $\frac{1}{10}$ inch on a side. Usually every tenth line and sometimes every fifth line is made a trifle heavier than its neighbors, so that the number of divisions may easily be counted when points are being plotted. Squared paper ruled in inches and eighths of an inch is useful for making sketches of machine parts to scale, but it is confusing when used for plotting points whose coördinates have decimal values.

52. Use of Squared Paper.—In using squared paper when plotting points, the first thing to do is to locate the positions of the coördinate axes. Any two lines crossing at right angles may be taken as the axes of reference; but usually two of the heavy lines are selected. If the values of the coördinates of the points to be plotted are all positive, so that the points all lie in the first quadrant, the origin, or point at which the axes cross, should be taken near the lower left-hand corner of the sheet. Most of the graphs used in engineering work deal with positive values and so the majority of diagrams have the origin at the lower left-hand corner. The left-hand vertical line is then the Y axis and the bottom horizontal line is the X axis. If the coördinates of the points are both positive and negative, the axes of reference should cross near the middle of the sheet, so that the abscissas and ordinates may be laid off above, below, to the right, and to the left.

53. To illustrate the use of squared paper, let it be required to plot the following points: 5, 9; -6, -3; -2, 8; and 4, -7. As the coördinates have both positive and negative values, two of the heavy intersecting lines at the middle of the sheet of squared paper are chosen as axes, as shown in Fig. 24, and each square is assumed to have a value of 1.

The first point, 5, 9, lies in the first quadrant, because its coördinates are both positive. So, 5 squares are counted off to the right from O and then 9 squares upwards from the X axis, locating the point a, which is the position of the first point.

The coördinates of the second point are both negative, indicating a position in the third quadrant. Hence, six squares are counted off to the left from O and then 3 downwards from the X axis, locating the second point, b.

The abscissa of the third point is negative and the ordinate is positive; therefore, the point $-2, 8$ lies in the second quad-

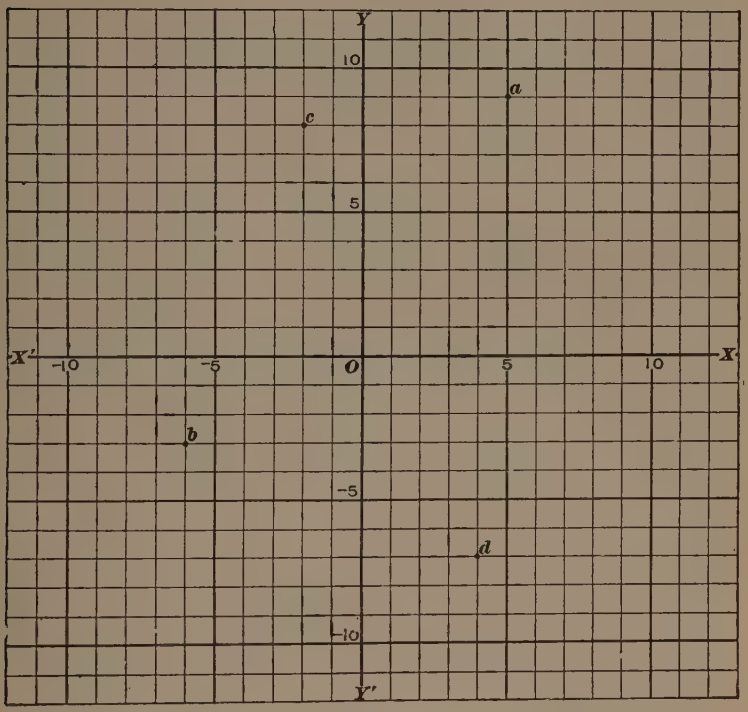

Fig. 24

rant. From O count off two squares to the left and then 8 squares upwards from the X axis, locating the point c.

The last point lies in the fourth quadrant, because the abscissa is positive and the ordinate is negative. From O are counted off 4 spaces to the right and 7 downwards from the X axis, locating the point d.

EXAMPLES OF GRAPHS

IRREGULAR GRAPHS

54. Constants and Variables.—If a quantity always has the same value, it is called a **constant.** Thus, the number of inches in a foot is a constant, as its value is always 12; the number of cubic inches in a liquid gallon is a constant, as it is 231; the number of pounds in a short ton, or 2,000, is a constant; and there are many others. But if a quantity has no fixed value, so that it varies according to conditions, it is called a **variable.** Thus, the temperature of the air is a variable, because it changes from hour to hour or from day to day; the speed of an electric car is a variable; and there are many other examples that might be given.

If two variable quantities are so related that when the value of one changes, the value of the other also changes, then one is said to be a function of the other. Thus, the sine of an angle is a function of the angle, because, for every value the angle may have, there is a corresponding, yet different, value of the sine; that is, the sine changes when the size of the angle changes.

The relation between two variable quantities can always be indicated by a graph. If the value of one depends on the value of the other, that is, if one is a function of the other, the graph will be regular in its outline; but if the value of one may vary without regard to the value of the other, the curve or graph will be irregular.

55. Example of Irregular Graph.—The graph shown in Fig. 20 is an excellent example of an irregular graph. The voltage and the time are absolutely independent of each other. The voltage may have any value at any time; consequently, the curve is a succession of peaks and valleys, or ridges and hollows, with no order or regularity of arrangement. As a further example, take the case of a train starting from a station and running for 40 minutes, the speed at various intervals being noted. Suppose that the time, in minutes, elapsed from the

moment of starting, and the corresponding speeds in miles per hour, are as follows:

Time Elapsed Minutes	Speed in Miles per Hour	Time Elapsed Minutes	Speed in Miles per Hour	Time Elapsed Minutes	Speed in Miles per Hour
1	21	15	29	29	30
2	30	16	27	31	31
4	$30\frac{3}{4}$	18	25	32	33
6	28	20	18	34	33
7	24	21	18	35	34
8	32	22	20	37	32
10	22	23	26	38	31
11	27	25	27	39	30
13	30	26	26	40	28

The speed of the train is independent of the time; therefore, a graph showing the relation between these two variables will be an irregular graph.

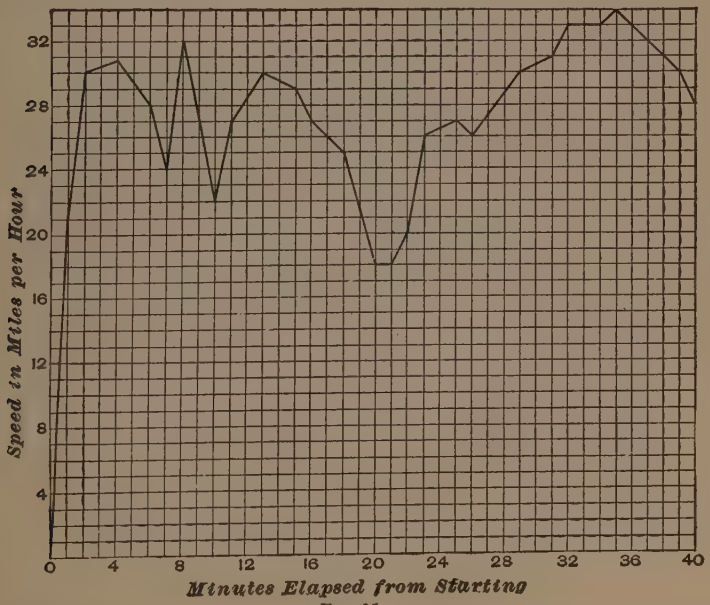

Fig. 25

56. The graph showing the relation of the values in the preceding article is given in Fig. 25. Each pair of values, as,

for example, 1 and 21, 2 and 30, 4 and $30\frac{3}{4}$, etc., are the coördinates of a point on the curve. For convenience in plotting, each small square is assumed to represent one minute of time and one mile per hour of speed. The values of time elapsed are taken as abscissas and are laid off horizontally to the right from the origin, and the speeds are taken as ordinates and are laid off upwards from the X axis. The positions of the successive points are indicated by dots, and these dots, when joined by straight lines, give the graph. As predicted in the last sentence of the preceding article, the graph is irregular.

Irregular graphs will also be obtained by plotting the various pressures existing in a steam boiler at intervals throughout a given time, the quantity of water flowing past a certain point in a stream during a period covering half a year, the values of stocks and bonds from day to day in a changing market, the output of a factory at successive periods of time during the year, and many other similar cases.

REGULAR GRAPHS

57. Dependent Variables.—In the case of the graph just described, the time and the speed had no definite relation; that is, these two variables were independent of each other, and the graph was irregular as a result. If two variables are dependent, that is, if they are so related that a change in the value of one produces a corresponding change in the value of the other, the graph showing their relation will be regular. In other words, there will be no haphazard changes of direction of the curve; instead, the graph will be smooth and continuous. It may take the form of a straight line; but if it is a curve, the change of direction of the curve will be regular and there will be no abrupt peaks and valleys like those in Figs. 20 and 25.

58. Straight Graphs.—If two variable quantities are so related that a change in the value of one produces a change *of the same proportion* in the value of the other, the graph showing this relation will be a straight line. This may be very

easily illustrated by the example of an ordinary spring scales like that shown in Fig. 26. It consists of a coiled spring, one end of which is fastened to the upper end of the case *a* and the other end of which is connected to the bar that carries the hook *b*. A pointer *c* projects through the slot *d* in the front of the case and moves down over a graduated scale *e* when a weight is hung on the hook. The position of the pointer on the scale indicates the weight that is suspended from the hook. The amount that the spring stretches under the pull of the weight on the hook is directly proportional to the weight. That is, a weight of 2 pounds will stretch it twice as far as a weight of 1 pound, and a weight of 10 pounds will stretch it 10 times as far as a weight of 1 pound. In other words, the stretch varies in the *same* ratio as the weight. Therefore, a graph showing the relation will be a straight line.

Fig. 26

59. Suppose, for example, that a test is made of the scales shown in Fig. 26 and that the weight on the hook is gradually increased by the addition of 1 pound at a time, and that the stretch of the spring is observed after each increase of weight. Further, suppose that the spring is of such strength that it stretches $\frac{1}{8}$ inch for every pound of load. Then the results of the test will show the following:

Weight Pounds	Stretch Inch	Weight Pounds	Stretch Inches	Weight Pounds	Stretch Inches
1	$\frac{1}{8}$	9	$1\frac{1}{8}$	17	$2\frac{1}{8}$
2	$\frac{1}{4}$	10	$1\frac{1}{4}$	18	$2\frac{1}{4}$
3	$\frac{3}{8}$	11	$1\frac{3}{8}$	19	$2\frac{3}{8}$
4	$\frac{1}{2}$	12	$1\frac{1}{2}$	20	$2\frac{1}{2}$
5	$\frac{5}{8}$	13	$1\frac{5}{8}$	21	$2\frac{5}{8}$
6	$\frac{3}{4}$	14	$1\frac{3}{4}$	22	$2\frac{3}{4}$
7	$\frac{7}{8}$	15	$1\frac{7}{8}$	23	$2\frac{7}{8}$
8	1	16	2	24	3

If, now, the weights are taken as abscissas and the amounts of stretch as ordinates, and a graph is drawn, as shown in

Fig. 27, it will be a straight line. The reason is that the stretch increases at the same rate as the weight on the spring. In plotting the points through which the line of the graph is drawn, each square was assumed to represent 1 pound in a horizontal direction and $\frac{1}{4}$ inch in a vertical direction; so, to mark off $\frac{1}{8}$ inch, it is necessary to estimate, by eye, half the height of a square.

60. Selection of Scales.—In constructing a graph it is important to select suitable scales for the values that are to be plotted. In Fig. 27, for example, the values are small, and it is convenient to let each small square represent $\frac{1}{4}$ inch of stretch of spring and 1 pound of weight. But if the spring had been

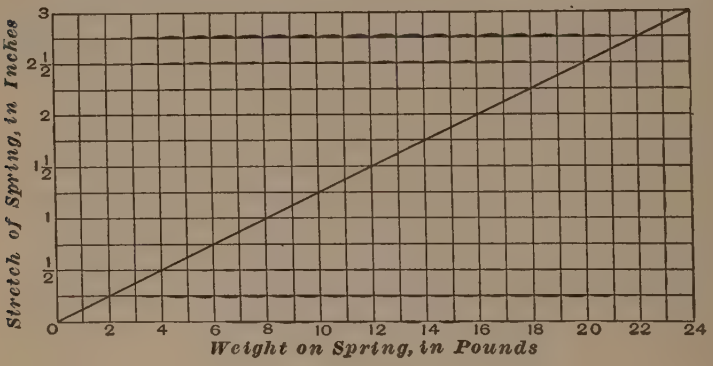

Fig. 27

large and heavy, so that perhaps 1,000 pounds would have been required to produce $\frac{1}{4}$ inch of stretch, the same scale of weights could not have been used, as it would have made a graph so long that it could not have been shown on a sheet of any practicable size. Therefore, if the graph is to show the relation between values that are large, it is necessary to choose scales such that the resulting diagram will be of convenient size. It is impossible to give a specific rule for finding the scales to be used in all cases. The main point to be observed is to choose scales such that the plotted points will be far enough apart to avoid confusion. The number of points to be plotted, the accuracy with which the graph is to be read, and the size of the

sheet of squared paper on which the graph is to be drawn are factors that affect the scales to be used.

61. Locating Axes and Marking Points.—In marking the values that represent the vertical and horizontal scales to which the graph is drawn, it is not necessary to take the origin, or crossing point of the axes, as zero, unless the values to be plotted begin at zero. Take Fig. 20, for example. Here the voltage values lie within the extremes of 325 and 500; hence, one of the horizontal lines near the bottom of the diagram is marked *300* and the other values are set down

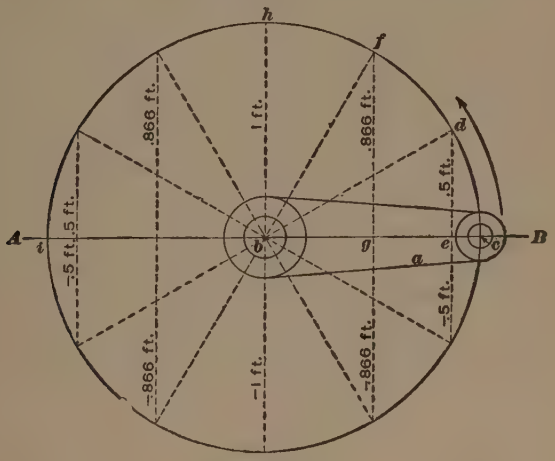

FIG. 28

above it, to a scale of 25 volts to each small square. If the scale of voltages had been begun at zero, there would have been a large amount of blank, waste space on the diagram. Therefore, if the values to be plotted do not begin at zero, any convenient vertical and horizontal lines may be taken as the axes of reference in place of those that intersect at zero.

In the work of plotting, the positions of the points may be indicated by dots on the squared paper. If the paper is ruled closely and dots are hard to see, a circle may be drawn around each, thus, ⊙; or, if desired, a small cross, ×, may be drawn so that its arms intersect at the point to be located.

62. Curved Graphs.—The crank *a*, Fig. 28, is supposed to rotate around the shaft *b*, so that the center of the pin *c* swings in a circle, as shown. If the crank makes one complete turn, in the direction shown by the arrow, and comes back to its original position, it will swing through an angle of 360°, or four right angles, because one complete circle contains 360°. Now, suppose that the positions of the crank for every 30° of turning are drawn, as shown by the dotted lines running out from the center *b*. When the crank has moved 30°, the center of the pin *c* will be at *d*. Assume that the distance from *b* to *c* is exactly 1 foot. Then, when the center of the pin is at *d*, it will be .5 foot above the line *A B*; that is, *d e* will measure .5 foot.

This must be true, because the crank has moved through 30°, and the sine of 30° is .5; that is, *d e* divided by *b d* is equal to .5, and as *b d* = *b c* = 1 foot, it follows that *d e* = .5 × 1 foot = .5 foot. When the crank has swung through another 30°, or 60° in all, the distance *f g* from the center of the pin to the horizontal line *A B* will be .866 foot. Here again the distance is found by multiplying the length of the crank by the sine of the angle, which is .866 for an angle of 60°. When the crank swings to the position *b h*, it is at right angles to *A B* and the distance *h b* is 1 foot, which is equal to the length of the crank multiplied by the sine of 90°, or 1.

63. As the crank continues to swing beyond the position *b h*, Fig. 28, the pin approaches the line *A B* again, and when the pin reaches the point *i*, it is on the line *A B*, as at the start, and the distance from its center to this line is zero. When it swings below the line *A B*, the distances are the same as before, and in the same order; but, as they are measured *downwards* from the line *A B*, they are marked with a minus sign, to indicate that they are negative, or *below* the reference axis. This distance from the center of the pin to the line *A B*, measured in a vertical direction, is called the **displacement** of the pin. If the displacements are marked down in order, with the corresponding angles through which the crank has swung, the following list will result:

Angle Degrees	Displacement Foot	Angle Degrees	Displacement Foot
0	0	210	− .5
30	.5	240	− .866
60	.866	270	−1.0
90	1.0	300	− .866
120	.866	330	− .5
150	.5	360	0
180	0		

64. Now, let the relation between the angles and the displacements in the preceding list be shown by means of a graph, as in Fig. 29. As the displacements above and below the horizontal are positive and negative, the horizontal axis of reference for the graph is drawn at the middle of the diagram, and at the left-hand end is drawn the vertical axis of reference, the point of intersection being marked zero. The scale of displacements, which are marked on the vertical axis, is such that each square represents .1 foot, and the values are laid off both above and below the horizontal axis. The scale of angles is laid off to the right on the horizontal axis, each square representing 12°. In other words, angles are abscissas and displacements are ordinates. At 0° the displacement is zero, and so the first point a lies at the intersection of the axes. At 30° the displacement is .5 foot; so, as one square represents 12°, it is necessary to count off $\frac{30}{12}=2\frac{1}{2}$ squares along the horizontal axis, and then upward 5 squares to indicate .5 foot, thus locating the point b. An angle of 60° is represented by $\frac{60}{12}=5$ squares and .866 foot is laid off above this. As each square vertically represents .1 foot, a distance of .866 foot is equal to $\frac{.866}{.1}=8.66$, or a little more than $8\frac{1}{2}$ squares, and thus the point c is located. In a similar manner the other points are located, the negative displacements being laid off below the horizontal axis.

65. After the various points have been plotted, as indicated by the dots in Fig. 29, a smooth curve is drawn so as to pass

through the points *a, b, c,* etc., in regular order. This curve is the graph that shows the relation between the angular movement of the crank and the distance of the center of the crank-pin from the horizontal line *A B,* Fig. 28. It is known as the **sine curve,** or **curve of sines,** so called because, as explained in Art. **62,** the vertical distance from the center of the pin to the line *A B* is the sine of the angle made by the crank with that line, assuming the length of the crank to be **1.** It is a regular graph, as its changes of curvature are never abrupt, but gradual.

If the crank were longer or shorter than 1 foot, the displacement for any position of the crank would equal the product of

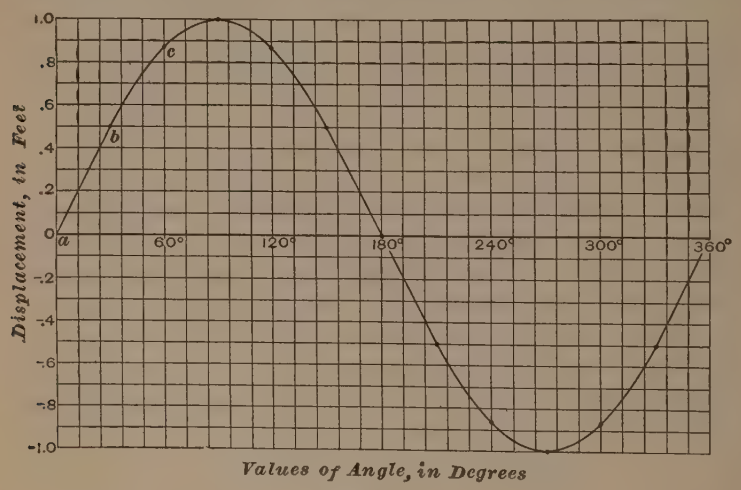

Fig. 29

the length of the crank and the sine of the angle through which the crank has turned from the horizontal position.

66. Polar Graphs.—The graphs thus far considered are made up of curves or lines drawn through a series of points whose locations are determined by reference to two axes of reference at right angles to each other. In the graph shown in Fig. 30, however, the points are located by **polar coördinates,** which are (*a*) the distance of the point from a center called the *pole,* and (*b*) the angle made by a horizontal line

through the pole and a line joining the point and the pole. In the illustration, O is the pole. It is surrounded by a series of concentric circles spaced at equal distances. These are numbered from the pole outwards in regular order, beginning with 1; thus, a point anywhere on the circle 1 is at unit distance from the pole; a point on the circle 6 is at a distance of 6 units from the pole; and so on. The unit distance may represent an inch, a foot, or any other unit of measurement. Similarly, a

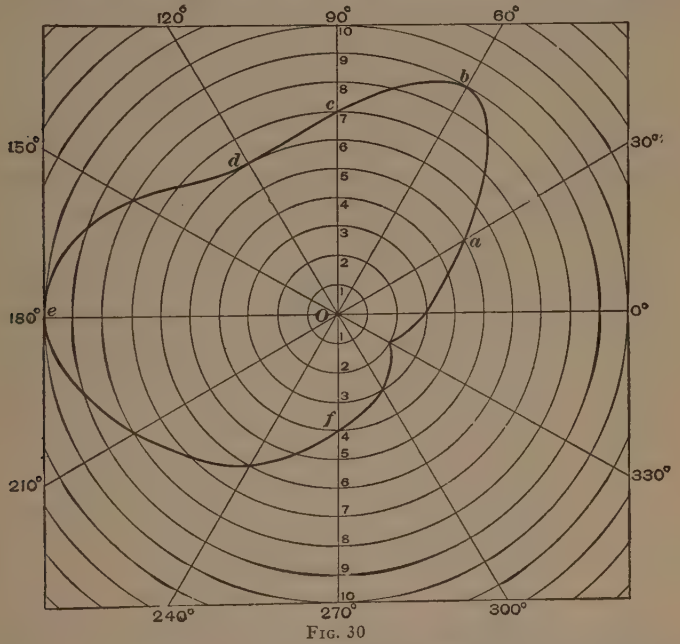

FIG. 30

series of radial lines are drawn through the pole, making a number of equal angles of 30°, as shown. The right-hand end of the horizontal line is taken as zero, and the angles are marked with their values in order around the circle, in a direction counter-clockwise, or opposite to the direction of motion of the hands of a clock.

67. The position of any point on the diagram, Fig. 30, is determined by stating its distance from the pole and the angle made between the horizontal through O and the radial line on

which it lies. For example, suppose that the numbers on the circles represent feet. Then, let it be required to locate a point whose polar coördinates are 5 feet and 30°. As the distance from the pole is 5 feet, the point must lie on the circle *5*; and as the angle is 30°, it must lie on the radial line marked *30°*; therefore, the point must be at the point *a*, where the circle *5* crosses the radial line marked *30°*. The coördinates of the point *b* are 9 feet and 60°; of *c*, 7 feet and 90°; of *d*, 6 feet and 120°; of *e*, 10 feet and 180°, of *f*, 4 feet and 270°; and so on for other points.

Many recording instruments used in power plants to record steam pressures, flow of water, voltage of electric current, etc., employ circular charts that rotate at uniform speed and are divided into angles representing the hours and minutes during a day of 24 hours. A pen, whose distance from the center of the chart depends automatically on the steam pressure, flow of water, or voltage, as the case may be, marks a line as the chart rotates and so gives a graph that shows the value of the pressure, flow, voltage, or whatever it may be, at all times during the day. The principle is exactly that of the polar graph just described.

APPLICATIONS OF GRAPHS

68. Temperature Conversion Diagram.—The relation between the readings of the Fahrenheit thermometer and the centigrade thermometer may be shown conveniently by a graph, as in Fig. 31. On the Fahrenheit scale the freezing point is 32° and on the centigrade scale it is 0°; therefore, the first point *a* is located at the point representing 32 on the horizontal scale and 0 on the vertical scale. The boiling point on the Fahrenheit scale is 212 and on the centigrade scale is 100; hence the point *b* represents the relation of these values. A straight line is then drawn through *a* and *b*. The graph thus formed may be used to convert Fahrenheit temperatures to centigrade, or centigrade temperatures to Fahrenheit. Suppose, for example, that a temperature of 80° C. is to be converted into Fahrenheit temperature. Locate 80 on the vertical

scale, follow horizontally to the graph at *c*, then vertically to the bottom line at *d*, which point indicates 176 on this scale; therefore, 80° C. = 176° F. The reverse order is followed in converting Fahrenheit temperature to centigrade.

69. Efficiency-Speed Diagrams.—Sometimes it is convenient to plot two or more graphs on the same sheet. In Fig. 32, for example, is shown a diagram with two graphs, one indicating the efficiency and the other the speed of an electric motor of a certain type in relation to the current supplied to it. As there are two curves, or graphs, there must be two vertical

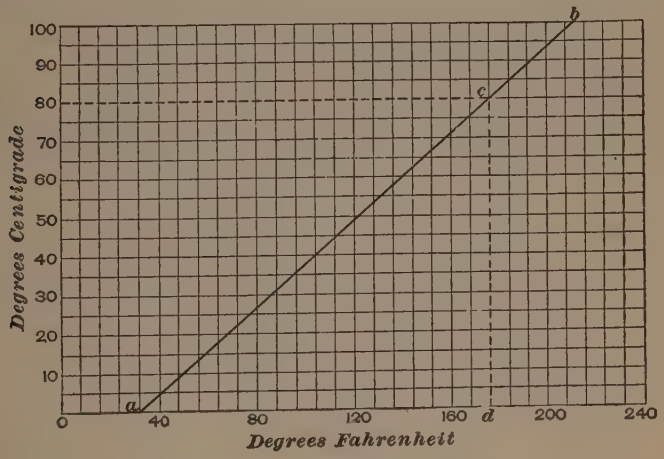

Fig. 31

scales. So, to prevent confusion, one scale is placed at the left-hand edge and the other at the right-hand edge, and each is clearly marked so that it can be used in connection with its proper graph; that is, the efficiency-curve values are read on the right-hand scale and the speed-curve values on the left-hand scale.

It will be observed that the speed curve drops steadily from the left toward the right. This indicates that the speed of the motor decreases rapidly at first and then less rapidly, as the current is increased. The efficiency graph first curves upwards slightly and then downwards. This indicates that the efficiency

rises, or increases, slightly as the current is increased, up to about 75 amperes, when the highest value is reached. Beyond that point it decreases gradually, as indicated by the slow falling of the curve.

70. The diagram in Fig. 33 contains three curves, showing the relation of speed, efficiency, and current to horsepower in the case of an electric motor. Each curve is marked clearly, so as to indicate to what it refers. Instead of writing the vertical scales at opposite sides of the diagram, they are both

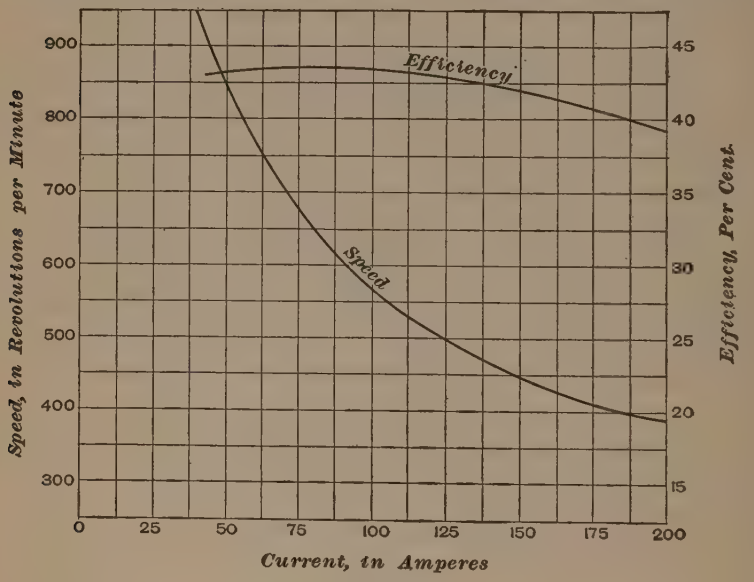

FIG. 32

placed at the left-hand edge, with lines of explanation to indicate what they mean. The first scale at the left is used for reading the values of points on both the speed curve and the efficiency curve, and the other one is used in connection with the current curve. The directions in which the curves slope indicate that, as the horsepower increases, the speed steadily decreases, the efficiency rises rapidly at first and then falls off, and the current increases at a rate that gradually grows more rapid.

If one of the two variables changes at a different rate from the other, the graph will curve upwards or downwards, depending on whether the rate of change of the first is greater or less

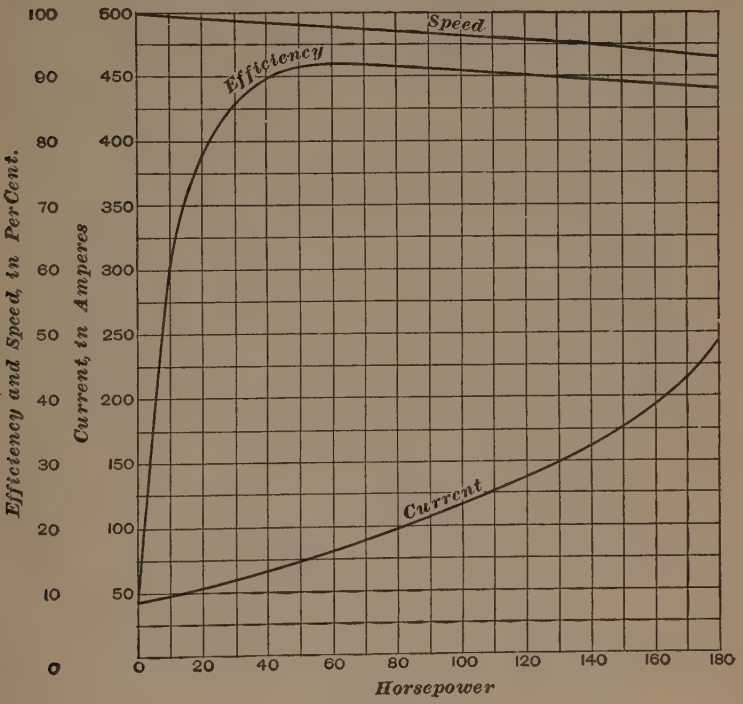

Fig. 33

than that of the other. If the rate of change in both is the same, the graph will not curve at all; that is, it will become a straight line.

TABLES

OF

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

GIVING THE VALUES OF THE FUNCTIONS FOR ALL DEGREES AND MINUTES FROM

0° TO 90°

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94223	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71974	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infin.	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.473	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81812	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5 54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2 78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	40
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

This lesson on The Slide Rule is optional. In case you wish to take it up, you may do so at any time, but it will be necessary, before beginning the work, to provide yourself with a slide rule.

THE SLIDE RULE

Serial 1879 — Edition 1

CONSTRUCTION AND USE

ARRANGEMENT OF SCALES

INTRODUCTION

1. Usual Form of Slide Rule.—The slide rule is an instrument by means of which various mathematical calculations may be performed mechanically. It usually consists of a boxwood or mahogany body faced with celluloid strips on which the scales or graduations are marked. The ordinary slide rule, and the form most commonly used, is about 11 inches long, $1\frac{3}{4}$ inches wide, and $\frac{3}{8}$ inch thick. There are other sizes, some being as short as 5 inches and others 24 inches or more in length; but the size most convenient to handle is that first mentioned.

2. Descriptions of Slide Rules.—One form of slide rule is shown in Fig. 1, which illustrates the face of the rule. There are four scales lengthwise of the rule, and these are designated from top to bottom by the letters *A*, *B*, *C*, and *D*. The scales *A* and *B* are graduated alike; so are the scales *C* and *D*. But these two pairs of scales differ in the size and number of graduations. The scales *A* and *D* are fixed to the body of the rule and therefore they are always in the same relative positions with respect to each other. The scales *B* and *C* are fixed to a movable piece called the **slide**; thus, while they always have the same positions relative to each

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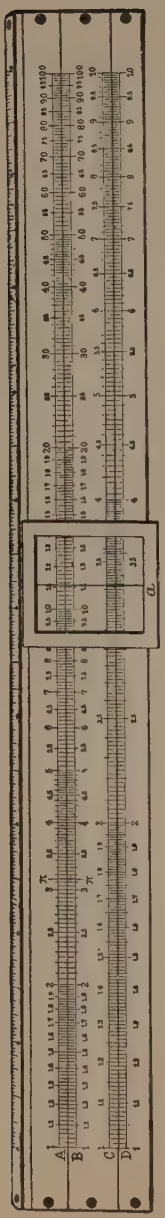

Fig. 1

other, they can be moved to right or left so as to assume any desired position with respect to the scales *A* and *D*.

3. Another form of slide rule is shown in Fig. 2. It has the four scales *A*, *B*, *C*, and *D*, just like the rule previously described; but in addition it has a fifth scale along the middle of the slide. This extra scale is graduated like the *C* and *D* scales, but in reverse order; in other words, it is the same as the *C* scale would be if it were turned end for end. As it is an inverted *C* scale, it is designated as the *C I* scale. The advantage of this inverted scale lies in reducing the number of settings required in performing certain calculations, as will be explained later.

Along the edge of the rule is a scale of the same length as the others, but graduated in three equal sections that are alike. This scale is called the *E* scale and its purpose, as explained more fully later, is to enable the cubes and cube roots of numbers to be found readily.

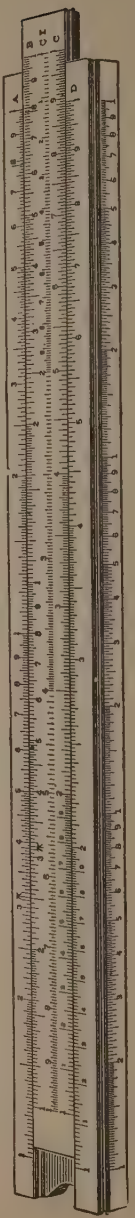

Fig. 2

4. Runner.—In the upper and lower edges of the body of the rule are lengthwise grooves into which fit the tongues of a metal frame *a*, Fig. 1, known

as the **runner,** and sometimes called a **cursor** or an **indicator.** It carries a plate of glass about an inch square, on the under side of which is a fine black line, or **hair-line,** as it is called. This line is at right angles to the scales on the rule. The runner may be moved to right or left over the face of the rule, and the purposes of the hair-line are to refer from one scale to another and to assist in reading results from the scales.

GRADUATIONS

5. Divisions of the Scales.—The upper scale *A*, Fig. 1, is in two parts that are alike in the number and spacing of the graduations; that is, the half of the scale from 1 to 10 is the same as the half from 10 to 100. Only the numbers marked on the graduations are different. The part from 1 to 10 is called the *left-hand A scale* and the part from 10 to 100 is called the *right-hand A scale.* The scale *B* is similarly divided, and its halves are known as the *left-hand B scale* and the *right-hand B scale.*

The ends of each scale are called the **indexes,** those at the left being the *left-hand indexes* and those at the right the *right-hand indexes.* The left-hand index of each scale is marked 1; but the right-hand indexes of scales *A* and *B* are marked 100, while the right-hand indexes of scales *C* and *D* are marked 10. In some makes of scales, all the indexes at both ends are marked 1 and the middle points of the scales *A* and *B* are marked 1, as, for example, in the rule shown in Fig. 2.

6. One of the first things noted in connection with the graduations, Figs. 1 and 2, is that no two adjacent divisions on a scale are equal. For instance, on the scales *A* and *B*, the distance between the points marked 1 and 2 is divided into ten main parts, no two of which are equal in length. Each of these ten parts is subdivided into five smaller parts, no two of which are equal. Yet each main division represents $\frac{1}{10}$ of the difference between 2 and 1, or $\frac{1}{10}$ of 1, or .1, and each small division represents $\frac{1}{5}$ of .1, or .02.

The portion of the C or D scale between the left-hand index 1 and the large figure 2 is similarly divided into ten main parts each having a value of .1. These parts are subdivided into ten smaller parts each representing a value of .01.

*7. **Logarithmic Basis of Graduation.**—The adjacent divisions on the scales of a slide rule are unequal because the scales are graduated to represent the mantissas of the logarithms of the numbers marked on the various divisions, and the mantissas do not bear the same ratio as the numbers themselves. For example:

log 1 = .00000	log 4 = .60206	log 7 = .84510
log 2 = .30103	log 5 = .69897	log 8 = .90309
log 3 = .47712	log 6 = .77815	log 9 = .95424

The logarithm of 10 is 1.00000.

Now, if the scale D were divided into 100,000 equal parts, and 30,103 of those equal parts were laid off from the left-hand index toward the right, the point marked 2 would be located; in other words, the left-hand index represents zero, or the logarithm of 1, and the distance from 1 to 2 represents $\frac{30,103}{100,000}$, or .30103, which is the mantissa of the logarithm of 2. Similarly, if 47,712 of the 100,000 equal parts were laid off to the right from 1, the point 3 would be located; 60,206 parts from the left would locate 4; and so on to the right-hand index. The distances of the lines marked 2, 3, 4, 5, 6, 7, 8, 9, and 10 from the left-hand index therefore represent the mantissas of the logarithms of those numbers.

The A and B scales are also divided on the logarithmic basis; however, each consists of two similar parts, each of which is half the length of the C or D scale. The length of each of these halves is taken to represent the logarithm of 10, and the graduations are so arranged that the distance of any number from the left-hand end of the half represents the mantissa of the logarithm of that number.

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*Although the slide rule is based on logarithms, a knowledge of this subject is not necessary for its use. Arts. **7** to **11**, inclusive, may therefore be disregarded by persons who are not familiar with logarithms.

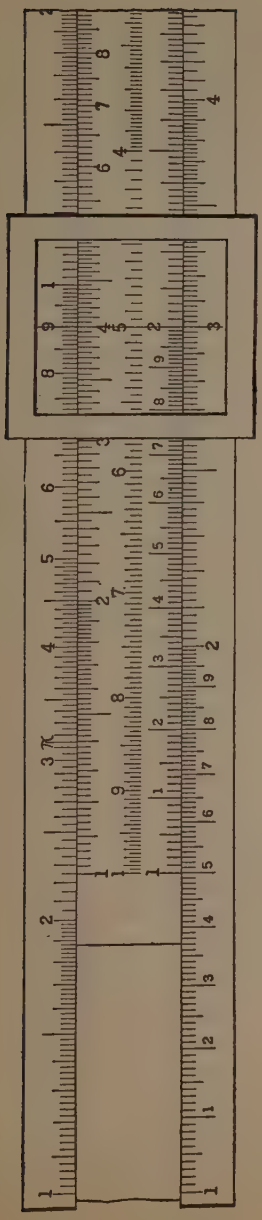

Fig. 3

The E scale, on the edge of the rule in Fig. 2, consists of three similar scales, each one-third the length of the C or D scale; and each part is divided logarithmically.

8. Principle of Operation of Slide Rule.—If the logarithms of two numbers are added, the sum represents the logarithm of the product of those numbers. It has been shown that the C and D scales are so divided that the distances of the various points from the left-hand index represent the mantissas of the logarithms of the numbers marked at those points. Therefore, to multiply two numbers by means of the slide rule, the distances representing their logarithms are placed end to end and the total length is found, which is then the mantissa of the logarithm of the product of the numbers. To illustrate, Fig. 3 shows how the product of 1.5 and 2 is found by the slide rule. The value 1.5 is located on scale D and the left-hand end of scale C is set to it. Then the value 2 is found on scale C, and the product, 3, is read from scale D in line with 2 on scale C. The explanation is simple. The distance from 1 to 1.5 on scale D is the logarithm of 1.5 and the distance from 1 to 2 on scale C is the mantissa of the logarithm of 2. By setting the slide as shown, these two distances are put end to end, and their total length is then the distance from 1 to 3 on

the *D* scale, which is the mantissa of the logarithm of 3. Hence, 3 must be the product of 1.5 and 2.

9. What has just been shown to be true of the product of 1.5 and 2 is equally true of the product of any two numbers found by the slide rule. For example, let the product of 6.5 and 8 be required. It is evident that this cannot be found by setting the left-hand index of scale *C* to 6.5 on scale *D*, and reading the product on scale *D* under 8 on scale *C*, because, when so set, 8 is beyond the end of scale *D*. However, if the right-hand index of the *C* scale is set to 6.5 on the *D* scale, then below 8 on the *C* scale will be found the product 52 on the *D* scale. In doing this, the distance representing the mantissa of the logarithm of 6.5 is first located on the *D* scale; from that distance is subtracted the entire length of scale *C*; and to the difference is added the distance representing the mantissa of the logarithm of 8; the total distance thus found is the mantissa of the logarithm of the product of 6.5 and 8. Since an entire scale length represents the logarithm of 10 (which is 1), it is obvious that its subtraction does not affect the mantissa of the logarithm of the product in any way. It is on such simple properties of logarithms that the various operations with the slide rule are based. Since the purpose of this Section is to teach the use of the slide rule rather than the theory involved, little stress will be laid on the reasons for the various operations that will be explained in the following pages.

10. Division is performed by the slide rule by subtracting the logarithm of the divisor from the logarithm of the dividend and finding the number corresponding to the difference. For instance, let 3 be divided by 2. The setting of the slide rule for solving this example is shown in Fig. 3. The point 3 is located on the *D* scale and directly above it is set 2 on the *C* scale. Then 1.5, which appears on the *D* scale opposite the index of the *C* scale, is the quotient. The explanation is as follows: On the *D* scale, the distance from 1 to 3 is the mantissa of the logarithm of the dividend 3. On the *C* scale, the distance from 2 to 1 (which is the same as from 1 to 2)

is the mantissa of the logarithm of the divisor 2. By placing the slide so that 2 on the *C* scale and 3 on the *D* scale coincide, the logarithm of 2 is subtracted from the logarithm of 3 and their difference is the distance from 1 on the *C* scale to 1 on the *D* scale. This distance is equal to that from 1 to 1.5 on the *D* scale, which is the logarithm of 1.5. Hence, $3 \div 2 = 1.5$, because $\log 3 - \log 2 = \log 1.5$.

11. When the slide rule shown in Fig. 2 is employed, multiplication or division may also be performed by using the *C I* and the *D* scales. However, since the *C I* scale is an inverted *C* scale, the operations are the reverse of those performed when the *C* and *D* scales are used. Thus, to multiply 3 by 2, set the runner so that the hair-line is exactly over 3 on the *D* scale. Move the slide so that 2 on the *C I* scale is exactly beneath the hair-line; in other words, set 2 on the *C I* scale directly above 3 on the *D* scale. Then, the product 6 is read on the *D* scale opposite the index of the *C I* scale, which is also the index of the *C* scale.

As an illustration of the use of the *C I* scale in performing division, let 6 be divided by 2. The right-hand index of the *C* scale is set to 6 on the *D* scale. The runner is then moved so that the hair-line is exactly over 2 on the *C I* scale, and directly beneath on the *D* scale is read the required quotient 3.

READINGS AND SETTINGS

12. Reading the Scales.—It is very important that the user of the slide rule be able to read values rapidly and correctly from the scales, and to set the hair-line of the runner or the index of the slide accurately to any desired value on the scales. The beginner will probably have more difficulty with these matters than with anything else connected with the use of the rule, but accuracy and speed can be attained by constant practice.

In order to determine the value indicated by any particular graduation other than those marked by figures, it is necessary to consider the number of divisions between the figure on the left and the figure on the right. A part of the *D* scale of

Fig. 1 is shown in Fig. 4, and the points marked *a*, *b*, *c*, etc. indicate positions at which readings are to be made from the scale. The point *a* lies between the left-hand index 1 and the main division marked 2, and the space between 1 and 2 is divided into 20 larger divisions, each containing 5 subdivisions, making 100 subdivisions in all. The difference between 2 and 1 is 1, and this length, or difference, is divided into 100 parts; therefore, each part represents $\frac{1}{100}$ or .01. As the graduation *a* lies three small divisions to the right of 1, the distance from 1 to *a* must have a value of $3 \times \frac{1}{100} = \frac{3}{100} = .03$; hence, the reading at *a* is $1 + .03 = 1.03$.

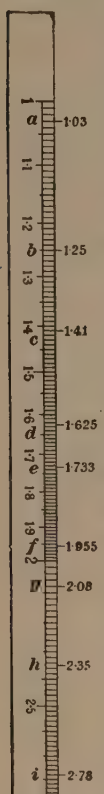

FIG. 4

13. It will be noted that the distance from 1 to 2, Fig. 4, is marked with small figures 1.1, 1.2, 1.3, etc. These values are located at the marks that are 10, 20, 30, etc. divisions from the end of the scale. As 10 divisions represent $\frac{10}{100} = .1$, the tenth mark to the right of 1 has a value of $1 + .1 = 1.1$. In the same manner, the twentieth division to the right of 1 corresponds to $1 + \frac{20}{100} = 1 + .2 = 1.2$; and so on for the remaining points 1.3, 1.4, etc. to 1.9.

The point *b* lies on the middle graduation between 1.2 and 1.3, and there are 10 divisions between these values. As $1.3 - 1.2 = .1$ and this difference is divided into ten parts, each part represents $\frac{1}{10}$ of $.1 = .01$. The point *b* is five small divisions to the right of 1.2 and so the distance from 1.2 to *b* corresponds to $5 \times .01 = .05$; therefore, the reading at *b* is $1.2 + .05 = 1.25$.

The point *c* is between 1.4 and 1.5 and between these two values are 10 divisions, each of which must therefore have a value of $\frac{1}{10}(1.5 - 1.4) = \frac{1}{10} \times .1 = .01$. As *c* lies one division to the right of 1.4, it represents a reading of $1.4 + .01 = 1.41$.

14. When the point at which a reading is to be found lies on a graduation, as in the preceding instances, the value is not hard to find. But, in using the slide rule, the readings must

often be determined at points that lie between graduations. For example, the point *d*, Fig. 4, does not lie on a graduation. Instead, it lies, as nearly as the eye can determine, midway between the second and third graduations to the right of 1.6, or, in other words, $2\frac{1}{2}$ divisions to the right of 1.6. As each division represents .01, the value at *d* is $1.6+(2\frac{1}{2}\times.01)=1.6+.025=1.625$.

The position of the reading at *e* must also be judged by eye. It lies between the third and fourth graduations to the right of 1.7, but closer to the third than to the fourth; in fact, it is about one-third of the distance from the third mark to the fourth. Consequently, the value of the reading is $1.7+(3\frac{1}{3}\times.01)=1.7+.03\frac{1}{3}=1.7+.033=1.733$.

The point *f*, judging by eye, lies about one-half a division to the right of the fifth graduation beyond 1.9, or $5\frac{1}{2}$ divisions to the right of 1.9. The reading at this point, therefore, is $1.9+(5\frac{1}{2}\times.01)=1.9+.055=1.955$.

15. The point *g*, Fig. 4, lies between 2 and 3. The distance between these two points, which represents $3-2=1$, is divided into 10 main parts, and each main part is further divided into five smaller parts. The distance between two adjacent main graduations, therefore, represents $\frac{1}{10}$ of 1, or .1, and each small division represents $\frac{1}{5}$ of .1, or .02. The point *g* lies between 2 and the first main graduation to the right of 2, which denotes 2.1; that is, *g* represents a value between 2 and 2.1. The point lies on the fourth small graduation to the right of 2. As each small division denotes .02, four such divisions denote $4\times.02=.08$; hence, *g* represents $2+.08=2.08$.

The position of the point *h* is also between 2 and 3. But it will be noticed that the fifth main graduation to the right of 2 is longer than any of the others. This line indicates the value 2.5. As the point *h* is to the left of it, the corresponding value at *h* must be greater than 2 and less than 2.5. By observing the main graduations, it is seen that *h* lies between 2.3 and 2.4 and that there are $2\frac{1}{2}$ small divisions between 2.3 and *h*. As each small division represents .02, it follows that

$2\frac{1}{2}$ such divisions represent $2\frac{1}{2} \times .02 = .05$; hence, h indicates $2.3 + .05 = 2.35$.

By a similar course of reasoning, the value at i is found to be 2.78 and at j is found to be 2.99.

16. Accuracy of Scale Readings.—The part of the C or D scale between 1 and 2 contains a greater number of divisions than any other part of any scale between two consecutive main graduations, and consequently readings or settings can be made with greater accuracy on this part than on any other part of any scale. But, from the examples that have been given, it is plain that no reading can be accurately made beyond four significant figures; and in most cases it is not possible to obtain an accurate reading to more than three significant figures. In any case, the fourth significant figure is obtained by estimating by eye the fraction of a subdivision. It is therefore only an approximation whose closeness to the true value depends on the skill and carefulness of the person reading the rule. In general, then, it may be said that the results obtained by the slide rule are not correct to more than three significant figures; but if a fourth significant figure can be approximated, that should be done.

17. In proceeding from the left to the right on the A, B, C, and D scales, the number of divisions between adjacent main graduations decreases. For example, on the C and D scales, there are 50 divisions between 3 and 4, but between 4 and 5, 5 and 6, 6 and 7, 7 and 8, 8 and 9, and 9 and 10, there are only 20 divisions. Thus, in the part of the C or D scale between 4 and the right-hand index, each subdivision has a value of .05 instead of .02 as between 2 and 4, or .01 as between 1 and 2. Similarly, on each half of the A and B scales, each division between 1 and 2 represents .02, between 2 and 5 represents .05, and between 5 and 10 represents .1. These differences must be kept in mind when the scales are being used, or confusion and error will result.

18. Same Setting for Same Significant Figures. Another important thing to be kept in mind is that, *in mak-*

ing a setting on the slide rule, the decimal point is ignored and only the significant figures and their order are considered. Thus, the hair-line on the runner would have the same position to indicate .00264, .0264, .264, 2.64, 26.4, 264, 2,640, or any other number in which those significant figures occur in the same order.

To those familiar with logarithms, the reason will be apparent if it is borne in mind that the slide rule is so graduated that the distance of a point from the left-hand index represents the mantissa of the logarithm of the reading at that point. Since the mantissa of the logarithm of a number is independent of the decimal point, the decimal point is ignored in making a setting on the slide rule.

Since settings are usually made to three significant figures, it is convenient to consider that the graduations on the various scales represent whole numbers of three figures. For example, on the *C* and *D* scales of the rule in Fig. 2, the left-hand index is called 100, the large figure 2 represents 200, the figure 6 between 1 and 2 indicates 160, the fourth main division between 2 and 3 is 240, etc.

19. In view of what has just been stated, it is necessary to add somewhat to the information given in Arts. **12** to **15.** It is true that the point *a*, Fig. 4, represents 1.03; but it also represents .103, 103, .0103, 10,300, etc. Similarly, the point *b* may denote .00125, 125,000, 12.5, etc., as well as 1.25. If the runner were to be set to 173.3, it would be moved until the hair-line stood at *e*, because 173.3 and 1.733 have the same significant figures. Again, the number 23,500 would be indicated by the point *h*, which indicates 2.35 equally well.

Of course, the decimal point must be considered in all calculations, if correct results are to be obtained. *But with the slide rule, the numbers are treated as whole numbers while the actual operations with the rule are being performed*, and when the significant figures of the answer have been obtained, the proper position of the decimal point is determined, as will be explained later.

20. Making Accurate Settings.—In multiplying or dividing by the aid of the slide rule, the index of the slide or the hair-line of the runner is set to certain given values. The

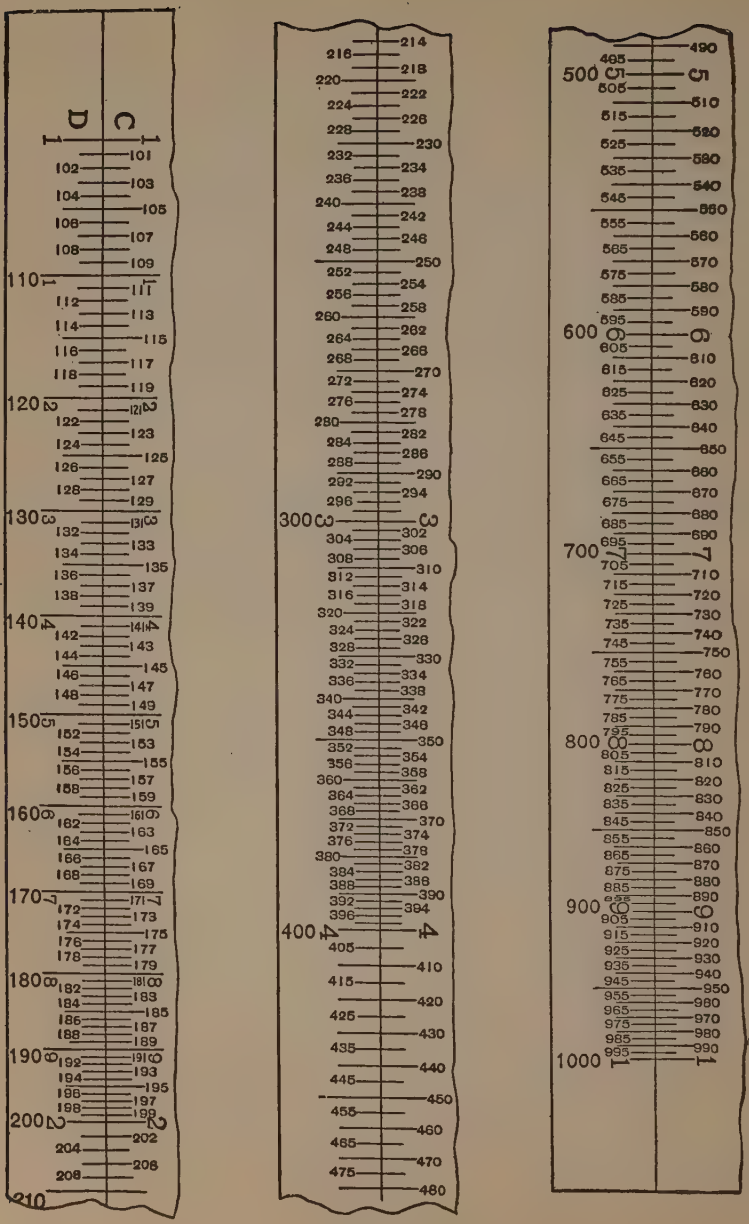
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FIG. 5

user of the rule should practice this feature of the work until he is able to set the hair-line or the index to any value, on any scale, with quickness and correctness. Expertness in the use of the slide rule is attained only by constant practice. To aid the beginner in determining the particular graduation that designates a given value, the values of all the graduations on the *C* and *D* scales are shown in Fig. 5. In order that the entire length of the rule may be given to a large scale in the illustration, it is shown broken into three sections. From what has been stated in Art. **18,** the division marked 635 not only indicates 635, but also .0635, 63.5, 6,350, or any other number whose significant figures are the same; and this is true of all other values shown.

21. The values shown in Fig. 5 are given to three significant figures, ending with 0 or 5 between 4 and the right-hand index. Consequently, to set the hair-line to some value that ends in a different figure, it is necessary to approximate. For example, let it be required to locate 49.7. This lies between 495 and 500—considering the significant figures and ignoring the decimal point; and as 497 is closer to 495 than to 500, the hair-line is set to the point *a*, Fig. 6. Again, let the number be 1,029. This number is between 1,020 and 1,030, which are indicated by 102 and 103 on the scale; also, 1,029 is very close to 1,030. Therefore, the hair-line is set to the point *b* to indicate 1,029. Suppose that 321 is the given value. This lies midway between 320 and 322. The value 320 is the second main graduation to the right of 3, and 322 is denoted by the first small graduation to the right of 320; hence, the point *c*, midway between 320 and 322, is the setting that indicates 321. Similarly, *d* indicates 166.6; *e*, 2,565; *f*, .0625; *g*, 791; *h*, .0403; *i*, 512.5; *j*, 56.25; *k*, .948; and *l*, 3.33.

22. In calculating by the slide rule, many of the numbers to be used will be found to have four or more significant figures, whereas, the settings and readings on the slide rule cannot be made accurately to more than three significant figures, except on certain parts of the scales. This being true, numbers having four or more significant figures should be

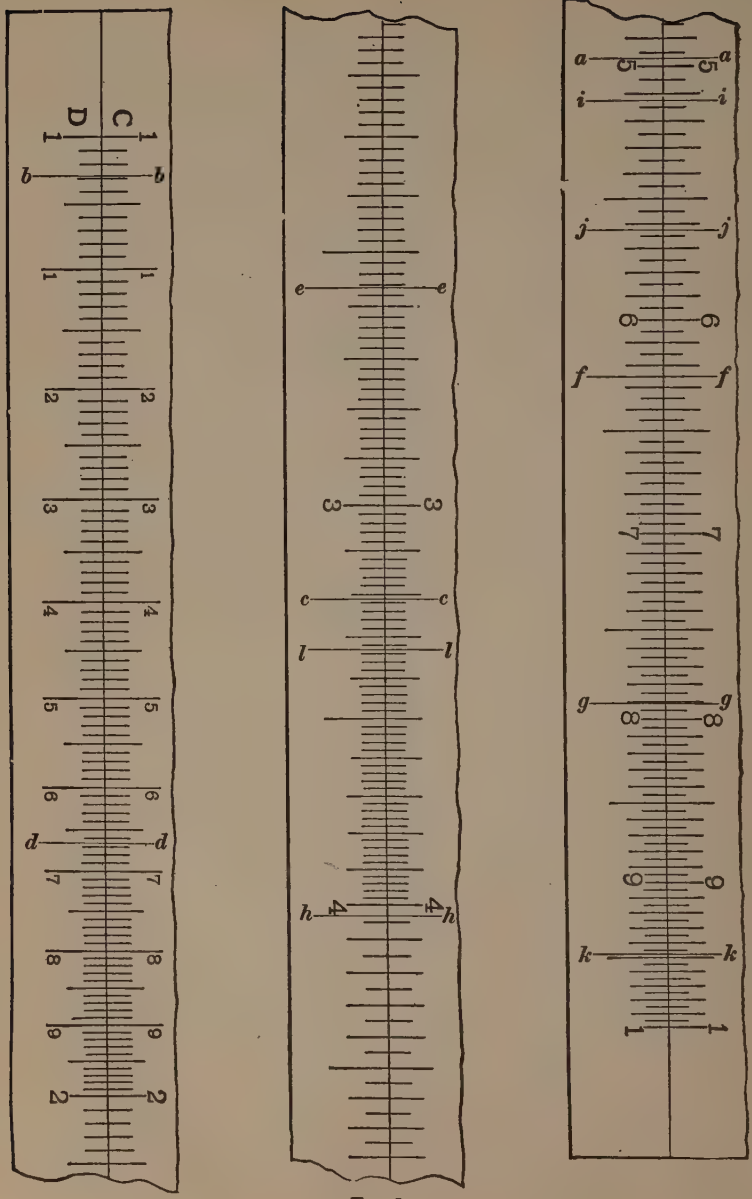


FIG. 6

altered to the nearest number having three significant figures, or fewer, except in those cases in which the fourth significant figure can be approximated in making the setting. Thus, the value 27,658 can not be located on the scale, and so it is taken as approximately 27,700. The value 277 may then be located very readily. If the number is 102,493, it may be taken as 102,500, and 1025 may easily be located on the scale. In the same way, 798,125 becomes 798,000; .0561375 becomes .0561; 45.083 becomes 45.1; .21448 becomes .2145, which can be located on the rule as the middle point between 2140 and 2150.

OPERATIONS WITH SLIDE RULE

MULTIPLICATION

23. As the *C* and *D* scales contain a greater number of divisions than the *A* and *B* scales between 1 and 10, it is usual to employ the *C* and *D* scales for multiplication and division, because they can be read accurately to a greater number of significant figures. However, the *A* and *B* scales may be used, if so desired.

To illustrate the use of the slide rule in performing multiplication, let the product of 25.6 and 33 be required. As already explained, the decimal point is ignored in making the setting. The left-hand index of the *C* scale is set to 256 on the *D* scale, as shown in Fig. 7. As 256 and 25.6 have the same significant figures in the same order, setting the index of the *C* scale to 256 on the *D* scale is equivalent to setting it to 25.6. Then, opposite 33 on the *C* scale will be found 845 on the *D* scale. Hence, the first three significant figures of the result are 845. The final step of the operation is to determine the position of the decimal point. This will now be explained.

24. There are various rules, many of them long and hard to remember, by which the decimal point or the number of figures in the answer may be found. But, if the slide rule is to

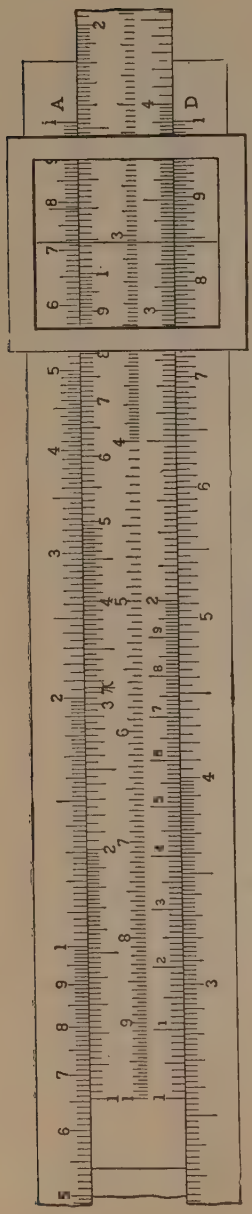


FIG. 7

save time in making calculations, it will be impossible to use such rules, because the time required to work out the desired result will be so great as to offset the time saved by using the slide rule. The simplest and quickest way, therefore, is to locate the decimal point by inspection and approximation. This is the method employed by all engineers who use the slide rule regularly; but accuracy and speed are attained only by continual practice. As a consequence, the beginner should set himself to the task of solving all sorts of examples, first performing the work with the slide rule and then checking his answers by the usual arithmetical operations, until he becomes so familiar with the methods to be used that his work will be swift and accurate.

25. The product of 25.6 and 33 was found by the slide rule to have the significant figures 845, but there was nothing to indicate whether the answer should be 84.5, 845, 8,450, or some other number containing the same significant figures. However, by approximating the two factors and performing a mental calculation, the number of figures in the answer may quickly be found. For instance, take 25.6 as approximately 25, and 33 as approximately 30. A *mental* multiplication shows at once that $25 \times 30 = 750$. In other words, the product of 25 and 30 is a whole number that

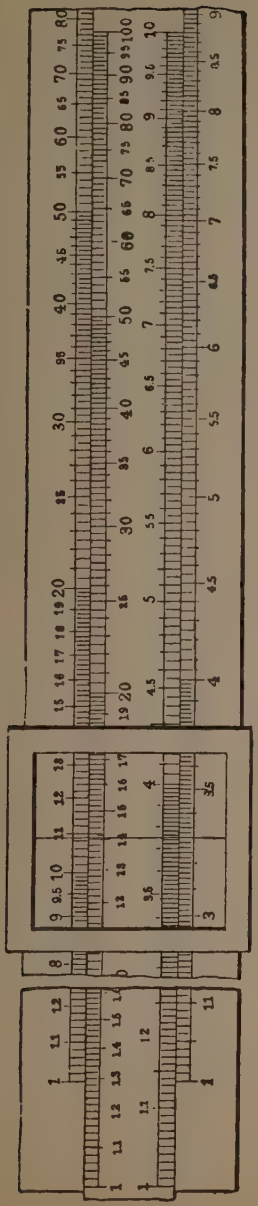


Fig. 8

contains three figures. Hence, the product of 25.6 and 33, which numbers are only slightly greater than 25 and 30, will also be a whole number containing three figures. The result as found from the scale *D* had the significant figures 845; hence, the product of 25.6 and 33 is 845. If the actual multiplication is performed by the usual arithmetical method, the product is 844.8, which shows that the slide rule gives a result quite close to the true result.

26. As a further example, let the product of .088 and 3.75 be required. In this case, if the left-hand index of the *C* scale is set to 88 on the *D* scale, the point 375 on the *C* scale will be outside the rule entirely. When such a case as this occurs, all that is necessary is to use the other index of the *C* scale. In other words, set the *right-hand* index of the *C* scale even with 88 on the *D* scale, as shown in Fig. 8, and opposite 375 on the *C* scale find 33 on the *D* scale. The points 88 and 375 are the settings for the two factors .088 and 3.75, since the decimal points are ignored, and the result, 33, found on the *D* scale, indicates that the first two significant figures of the answer are 33. The position of the decimal point, as before, may be found by approximating one or both of the factors and making a mental calculation. In this case, only one factor needs to be thus con-

sidered. It is evident that .088 is approximately .1, or $\frac{1}{10}$. If 3.75 is multiplied by $\frac{1}{10}$, mentally, the result is .375, which indicates that the decimal point precedes the first significant figure. Therefore, in the product of .088 and 3.75, the decimal point must precede the first significant figure of the result. The significant figures have been found to be 33, and if the decimal point is placed before the first figure, the answer becomes .33, which is then the product of .088 and 3.75.

27. From what has been given in the preceding articles, the following general summary of the operations required in multiplication may be made: Set the index (either the left-hand or the right-hand, as may be necessary) of the *C* scale to one of the factors on the *D* scale and opposite the other factor on the *C* scale read the product on the *D* scale. It makes no difference in which order the factors are taken. In Art. **23,** for instance, the example given was 25.6×33. It is evident that 25.6×33 has the same product as 33×25.6; therefore, the answer could just as correctly be obtained by setting the left-hand index of the *C* scale to 33 on the *D* scale and reading the product from the *D* scale opposite 256 on the *C* scale. Those familiar with logarithms will recall that in the slide-rule operation of multiplication the mantissas of the logarithms of the factors are added, and their sum must be the same, whichever factor is taken first.

28. The beginner in the use of the slide rule may have some difficulty at first in finding the position of the decimal point by the method given in connection with the examples in Arts. **23** and **25.** Until he gains experience and confidence, he may determine the correct result by observing whether the slide projects to the right or to the left in making the setting, and then using the following rule:

Rule.—*Treat both numbers as whole numbers and by the slide rule find the significant figures of their product; to these annex ciphers until the number of figures is equal to the sum of the numbers of figures, in both factors considered as whole numbers, if the slide projects to the left; but if it projects to the right, decrease the number of ciphers by 1. Finally, in this product,*

point off a number of decimal places equal to the sum of the decimal places in both original numbers.

This rule can be used for all problems in multiplication; but it should be abandoned in favor of the more rapid method stated in Arts. **24** and **25,** just as soon as the operator of the slide rule can acquire a fair degree of expertness in performing calculations.

29. Apply the foregoing rule to the example in Art. **26.** Considered as whole numbers, the factors are 88 and 375, and the significant figures of the result are 33, obtained by the setting shown in Fig. 8. The slide extends to the left, and so the product of the factors, considered as whole numbers, must contain as many figures as in both factors. As 88 has two figures and 375 has three figures, the product of 88×375 must have $2+3=5$ figures. So, ciphers are annexed to the significant figures 33 to make five figures in all, giving 33,000. Now, the original factors contain $3+2=5$ decimal places. If five decimal places are pointed off in 33,000, the result is .33000, or .33, which is the product of .088 and 3.75.

Consider the example in Art. **23.** As whole numbers the factors are 256 and 33 and the significant figures of the product are 845, as found by the setting in Fig. 7. The slide projects to the right and so the number of figures in the product of the factors, considered as whole numbers, is 1 less than the sum of the numbers of figures in both; that is, it is $(3+2)-1=4$. Annex a cipher to 845 to make four figures, and the product of 256 and 33 is 8,450. The original factors, 25.6 and 33, contain a total of 1 decimal place. So, point off one place in 8,450 and the result is 845.0, or 845, which is the product of 25.6 and 33.

When the rule shown in Fig. 2 is used, multiplication may be performed with the *D* and *C I* scales. By the aid of the runner, the multiplier on the *C I* scale is set over the multiplicand on the *D* scale, and the product is read on the *D* scale under the index of the *C* scale. For example, let the product of 25.6 and 33 be required. The decimal point is disregarded and the runner is moved so as to bring the hair-line over 256

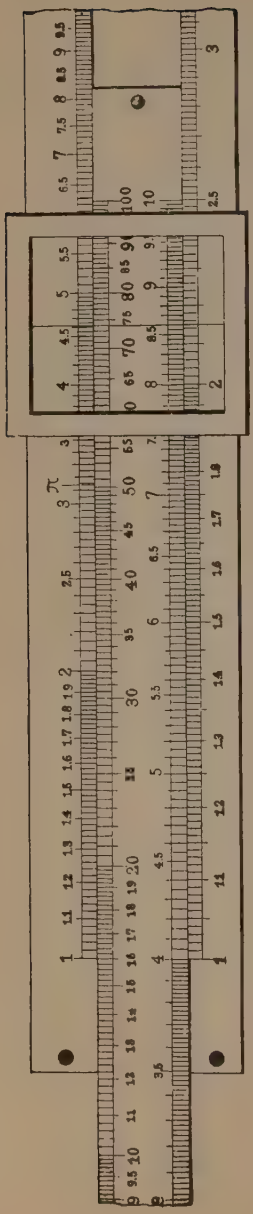


FIG. 9

on the *D* scale. The slide is then shifted to bring 33 on the *C I* scale under the hair-line, and under the index of the *C* scale is read the product 845.

When decimals are multiplied by using the *C I* and *D* scales it is best to determine the position of the decimal point by inspection, as in the examples of Arts. **25** and **26.** However, if it is desired to determine the position of the decimal point by applying a rule, the rule given in Art. **28** cannot be applied as it stands. But, if *right* is substituted for *left*, and *left* is substituted for *right* in the wording of that rule, then the rule will apply when the *C I* and *D* scales are used.

30. Thus far there have been only two factors to be considered; but, in very many problems, three or more numbers must be multiplied together. The principle is the same, but a greater number of settings of the slide are required. Suppose that the result of $.25 \times 8.6 \times .107 \times 75{,}186 \times 39.6$ is to be found. Ignoring the exact position of the decimal point in the factors, the values to which the settings on the rule must be made are 25, 86, 107, 75,186, and 396. The fourth number, 75,186, has five significant figures, and it has already been found that settings on the rule can be made accurately to only three significant figures, except in the case of a reading between 1 and 4 on the

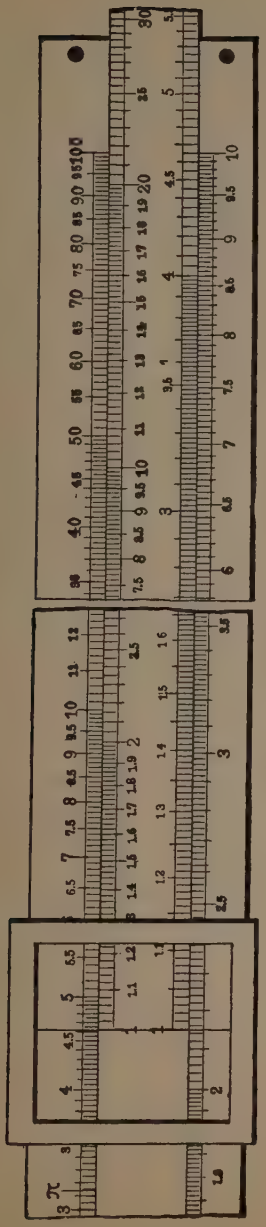


FIG. 10

C and D scales. Consequently, the number 75,186 is reduced to three significant figures by making it 752, which is the nearest approximate value having three figures. The five numbers involved in the settings are then 25, 86, 107, 752, and 396.

31. As shown in Fig. 9, the right-hand index of the C scale is set to 25 on the D scale and the runner is moved until the hair-line stands directly over 86 on the C scale. The point at which the hair-line crosses the D scale indicates the product of the factors .25 and 8.6, but this product is not read off, because further multiplications are to be made, and only the final product is desired; therefore, this product of the first two factors is merely marked by the position of the hair-line. Now, without disturbing the runner, move the slide until the left-hand index of the C scale is beneath the hair-line, as in Fig. 10, and then, leaving the slide stationary, move the runner until the hair-line is directly over 107 on the C scale, as in Fig. 11. The point where the hair-line crosses the D scale then indicates the product of .25, 8.6, and .107. As before, this product is not read, but is merely marked by the runner. Next, the runner is kept unmoved, and the right-hand index of the C scale is brought under the hair-line, as in Fig. 12. Then the slide is held stationary and the runner is moved until

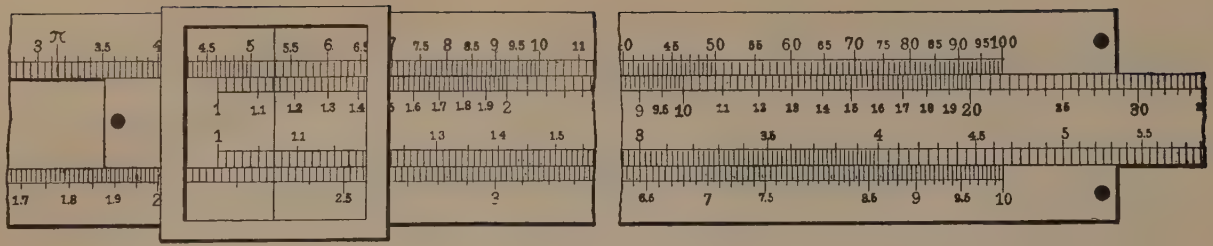


FIG. 11

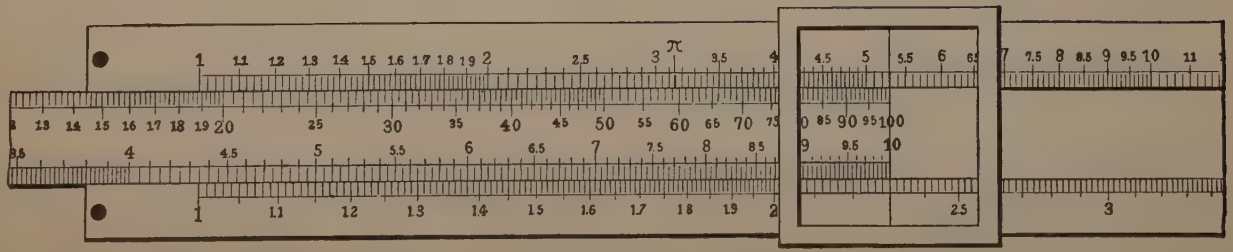


FIG. 12

the hair-line is over 752 on the C scale, as in Fig. 13. On the D scale it marks the product of the first four factors. Finally, with the runner stationary, the left-hand index of the C scale is brought under the hair-line, as in Fig. 14. Then the slide is held stationary while the runner is shifted to bring the hair-line over 396 on the C scale, as in Fig. 15. The point at which the hair-line crosses the D scale shows a reading of 685; that is, the product of $.25\times8.6\times.107\times75{,}186\times39.6$ is a number whose first three significant figures are 685. It remains to find the correct number of figures in the answer, or the position of the decimal point.

32. When there are several factors in the problem, as in the example given in the preceding article, it is not always easy to make a mental calculation to determine the position of the decimal point. In such a case it is better, in order to avoid errors, to approximate the values of the factors and perform a rapid cancelation with pencil and paper. The approximate values of the factors in the example just mentioned are as follows: $.25=\frac{1}{4}$; $8.6=9$; $.107=\frac{1}{10}$; $75{,}186=75{,}000$; $39.6=40$. If these are set down and the approximate result is found by cancelation, the work will appear as follows:

$$\tfrac{1}{4}\times9\times\tfrac{1}{10}\times75{,}000\times40=\frac{1\times9\times75{,}000\times\cancel{40}^{\cancel{10}}}{\cancel{4}\times\cancel{10}}=675{,}000$$

In other words, the product is a number of six figures preceding the decimal point; or, the result is a whole number of six figures. From the slide-rule operations the first three significant figures of the result were found to be 685; hence, to obtain six figures, ciphers are annexed, and the answer becomes 685,000, which is the product of $.25\times8.6\times.107\times75{,}186\times39.6$ according to the slide rule. As the true product is 684,942.95628, it is seen that the slide rule gives a very close answer.

33. The rule given in Art. **28** may be applied to a problem containing several factors, such as the one given in Art. **30.** These factors, considered as whole numbers, are

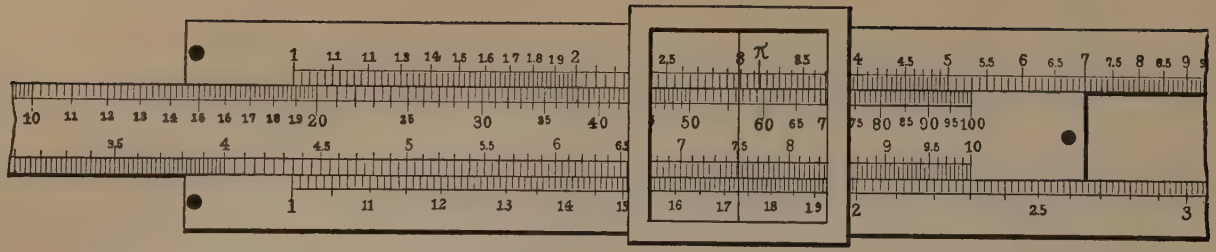


FIG. 13

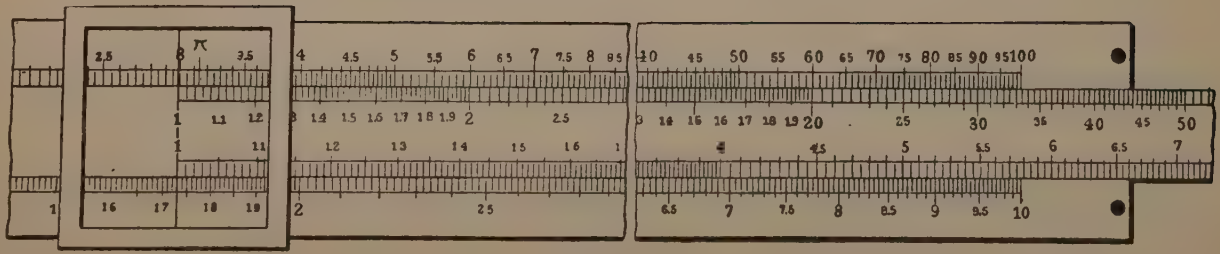


FIG. 14

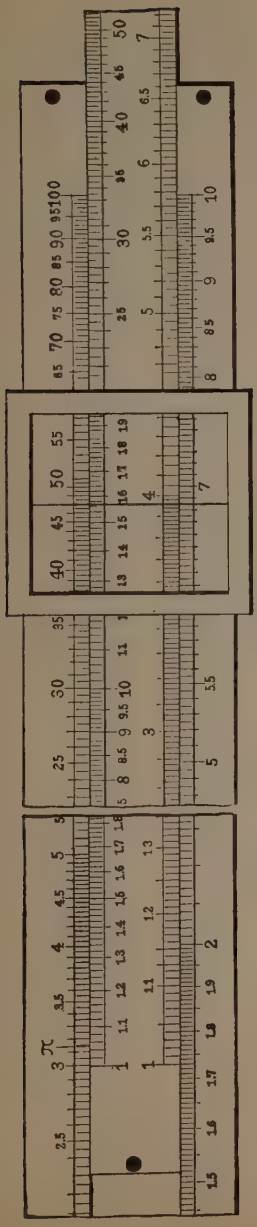


Fig. 15

25, 86, 107, 75,186, and 396, and the total number of figures in these numbers is 2+2+3+5+3=15. The significant figures of the result are 685, and if ciphers are annexed to make 15 figures, the number becomes 685,000,000,000,000. But, in the settings made in obtaining the product, the slide projected to the right twice. So the number of ciphers is reduced by two, giving 6,850,000,000,000. The total number of decimal places in the original factors is 2+1+3+0+1=7. Therefore, point off 7 places in the final number and the result will be 685,000.0000000, or 685,000.

34. When three or more factors are to be multiplied together, the type of rule shown in Fig. 2 is advantageous, because the *C I* scale may be used. By its use the product of three factors may be found with one setting of the slide, whereas the type of rule shown in Fig. 1 requires two settings of the slide for the same problem. To illustrate, take the example given in Art. **30.** The product of .25×8.6×.107 may be found with one setting of the slide. The method is to set one factor on the *C I* scale to the second factor on the *D* scale, using the hair-line on the runner, and then, under the third factor on the *C* scale, to read the product on the *D* scale. Thus, as shown in Fig. 16, 25 on the *C I* scale is set directly above 86 on the *D* scale, by aid of the hair-line,

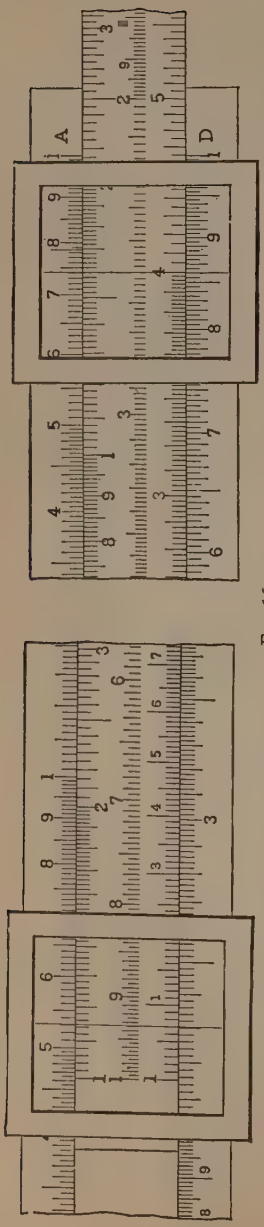


FIG. 16

and below 107 on the *C* scale the product is found on the *D* scale. As there are two more factors to be used, the product of the first three is not read, but is marked by moving the hair-line directly over it. The slide is then moved so as to bring 752 on the *C I* scale under the hair-line, and below 396 on the *C* scale the final product is read on the *D* scale. Thus, two settings of this type of rule accomplish the same end as several settings of the type shown in Fig. 1.

35. If the same number is to be multiplied successively by a number of different factors, the *C I* scale may be used to advantage. For instance, suppose that .434 is to be multiplied by 125, 150, 175, 200, 225, etc. up to 1,000. The runner is first set so that the hair-line stands over 434 on the *D* scale. Then the slide is moved until 1.25, 1.50, 1.75, 2.00, 2.25, etc. on the *C I* scale are brought in succession under the hair-line, and the product in each case is read from the *D* scale opposite the index of the *C* scale. Part of the time the reading will be found beneath the right-hand index of the *C* scale; but when the factors 250, 275, etc. are reached, the readings will be found opposite the left-hand index of the *C* scale.

36. In solving the example given in Art. **30,** the factors were used in the order stated in the example, but this is not the most advantageous

order. Obviously, the product will be the same, no matter in what order the factors are taken, but the amount of movement of the slide may be reduced by using the factors in the order of the size of their first significant figures, the largest first. If so arranged, the problem would be stated in this fashion: $8.6\times75{,}186\times39.6\times.25\times.107$. The advantage of rearranging the factors in this way will be seen at once if the multiplication is performed in the order given and the slide movements are compared with those made in the preceding solution.

EXAMPLES FOR PRACTICE

Find by the slide rule the product of:

(*a*)	3.7×462.	(*a*)	1,709
(*b*)	$4{,}211\times7{,}838$.	(*b*)	33,000,000
(*c*)	$1{,}728\times.065$.	(*c*)	112.3
(*d*)	18.17×124.	(*d*)	2,253
(*e*)	$287.5\times.302$.	(*e*)	86.8
(*f*)	$5{,}682\times543$.	(*f*)	3,085,000
(*g*)	$178.19\times1{,}004$. Ans.	(*g*)	179,000
(*h*)	$.000492\times4.1418$.	(*h*)	.002038
(*i*)	$.3685\times.042$.	(*i*)	.0155
(*j*)	$48\times375\times.0056$.	(*j*)	100.8
(*k*)	$.076\times25{,}486\times81.1\times360$.	(*k*)	56,600,000
(*l*)	$1{,}728\times8.5\times24\times36\times.0008375$.	(*l*)	10,630
(*m*)	$150\times3.5\times452.4\times118\times.0000303$.	(*m*)	849

DIVISION

37. When the *C* and *D* scales are used, division may be performed by reversing the operations of multiplication; that is, the divisor on the *C* scale is placed directly over the dividend on the *D* scale and the quotient is read from the *D* scale opposite the index of the *C* scale. The position of the decimal point in the result may be determined by inspection of the approximate values of the dividend and divisor, following the same method as in multiplication. For example, let the quotient of $26\div.152$ be required. As shown in Fig. 17, 152 on the *C* scale is set directly above 26 on the

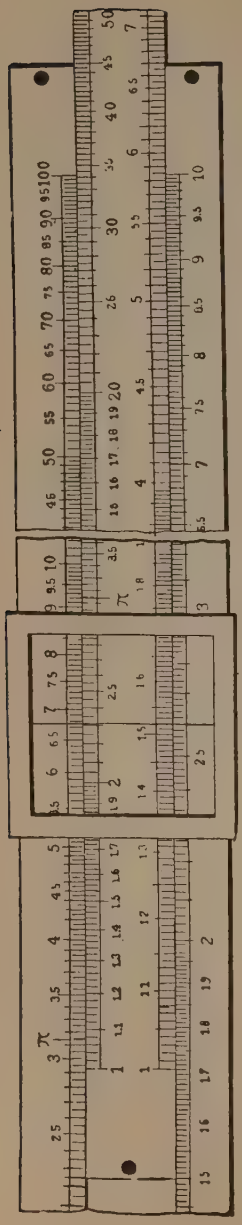


FIG. 17

D scale and the significant figures of the quotient are 171, found on the D scale below the index of the C scale. As .152 is approximately $\frac{1}{7}$, the quotient of $26 \div .152$ is approximately $26 \div \frac{1}{7} = 26 \times 7 = 182$. Hence, the quotient must be a whole number of three figures, and so 171 is the correct answer.

38. Suppose that the quotient of $5{,}280 \div 88$ is required. The setting of the slide rule, given in Fig. 18, shows that the significant figure of the result is 6. Now, 5,280 is approximately 5,300 and 88 is approximately 90, and $5{,}300 \div 90 = 60$, nearly. Hence, the quotient must contain two figures. The first is 6, according to the slide rule, so a cipher is annexed to make two figures, and the result is 60; that is, $5{,}280 \div 88 = 60$.

Again, take $16 \div 256$ as an example. The setting of the rule, shown in Fig. 19, gives 625 as the significant figures of the result. Assuming that 16 is approximately 20 and 256 is approximately 300, it is seen that $20 \div 300 = \frac{2}{30} = \frac{1}{15} = .07$, nearly. The quotient must therefore be a decimal, with the first significant figure two places to the right of the decimal point; that is, .0625 is the required answer.

39. As a further example, let .1564 be divided by 728. According to the setting of the rule, as shown in Fig. 20, the significant figures of the

quotient are 215. The two numbers, .1564 and 728, may be taken as .2 and 1,000, approximately. Then, it is seen at once that $.2 \div 1{,}000 = .0002$, and this result indicates that three ciphers intervene between the decimal point and the first significant figure. Hence, writing 215 so that three ciphers intervene between the decimal point and the first significant figure, the result is .000215, which is the quotient required.

40. There are rules by which the number of figures or the position of the decimal point may be found when using the slide rule for division; but, as stated in connection with the rule for multiplication, rules should be used only until sufficient familiarity is gained to enable the operator to determine the position of the decimal point by the more rapid method of approximating the values of the given numbers.

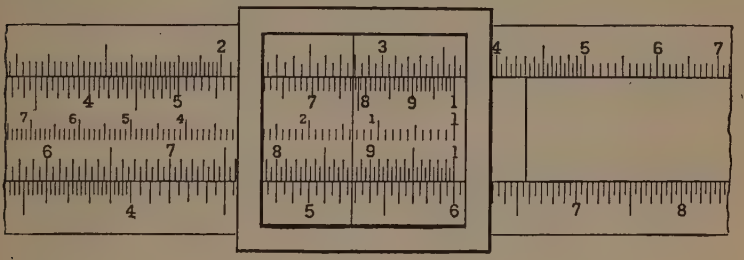


FIG. 18

In division, the given numbers may be whole numbers, mixed numbers, or decimals, and so several rules are required to fit the different cases that may arise. These cases are as follows:

1. Dividend greater than divisor, and both whole numbers.
2. Dividend smaller than divisor, and both whole numbers.
3. Dividend greater than divisor, and one or both containing decimals.
4. Dividend smaller than divisor, and one or both containing decimals.

41. When the dividend is greater than the divisor, and both are whole numbers, the number of figures to the left of

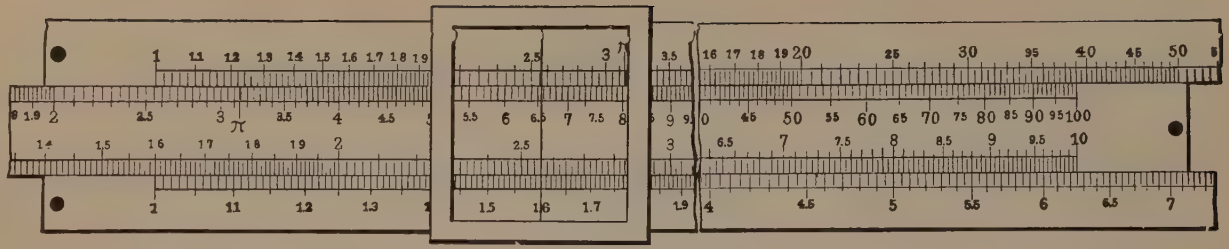


FIG. 19

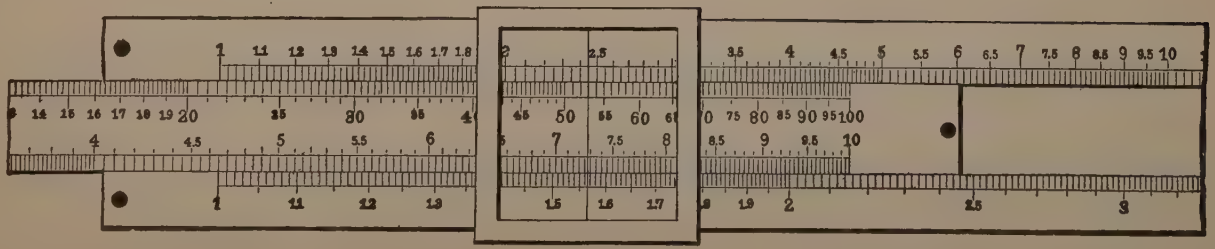


FIG. 20

the decimal point in the result may be found by the following rule:

Rule.—*If the quotient is obtained with the slide projecting to the left, the number of figures in the integral part of the quotient is equal to the number of figures in the dividend minus the number of figures in the divisor; but if the slide projects to the right, the number is 1 greater than the difference just mentioned.*

Apply this rule to the first example in Art. **38.** There are four figures in the dividend and two in the divisor, and the slide projects to the left, as may be seen in Fig. 18. Therefore, the number of figures preceding the decimal point in the quotient is $4-2=2$, and the answer is 60.

42. When the divisor is greater than the dividend, the quotient is less than 1, and the number of ciphers to be placed between the decimal point and the first significant figure of the answer is found by the following rule:

Rule.—*If the quotient is obtained with the slide projecting to the left, the number of ciphers between the decimal point and the first significant figure of the quotient is equal to the number of figures in the divisor minus the number of figures in the dividend; but if the slide projects to the right, the number is 1 less than the difference just mentioned.*

To illustrate this rule, consider the second example in Art. **38.** The dividend has two figures and the divisor has three, and the slide projects to the left, as shown in Fig. 19. So, the number of ciphers between the decimal point and the first significant figure is $3-2=1$; that is, .0625 is the quotient.

43. As in the division of integers, so, also, in the division of decimals, two cases may arise. The divisor may be either greater or less than the dividend. The method of procedure is as follows:

Rule.—*Treat both numbers as integers and find the result by the rule already given for the division of integers. Next, take the difference between the number of decimal places in the dividend and the number of decimal places in the divisor, and call this difference* **n**. *If the number of decimal places in the divi-*

dend exceeds the number of decimal places in the divisor, move the decimal point, in the result already found, **n** *places to the left; but if the number of decimal places in the dividend is less than the number of decimal places in the divisor, move the decimal point, in the result already found,* **n** *places to the right.*

From the statements just made, it follows that if the number of decimal places in the dividend is equal to the number of decimal places in the divisor, the result found by treating both as integers will be the required answer. There will be no necessity for altering the position of the decimal point, because the value of n in the rule will be zero. For example, if the quotient of $15.625 \div .375$ were required, the decimal points would be disregarded and 15,625 would be divided by 375, giving 41.7 as the answer.

44. To illustrate the foregoing rule, consider the example in Art. **37.** If both numbers are treated as whole numbers, the problem becomes $26 \div 152$, and, as shown in Fig. 17, the first three significant figures of the result are 171. According to the rule of Art. **42,** which is used because the dividend is smaller than the divisor, the number of ciphers between the decimal point and the first significant figure is $(3-2)-1=0$, because the slide projects to the right. That is, $26 \div 152 = .171$. But, the original divisor is .152, which contains three decimal places, and the dividend, 26, has no decimal places. Therefore, by the rule of the preceding article, the decimal point in the quotient just found must be moved three places to the right, making the answer 171, which is the quotient of $26 \div .152$.

45. As a further illustration, let the example of Art. **39** be used. If both numbers are treated as whole numbers, 1,564 and 728, the dividend will exceed the divisor and the rule of Art. **41** will apply. The setting of the slide rule is shown in Fig. 20, and the result thus obtained is 2.15. In the original numbers, however, there are 4 decimal places in the dividend and none in the divisor, the difference being $n=4--0=4$; and as the dividend contains the greater number of

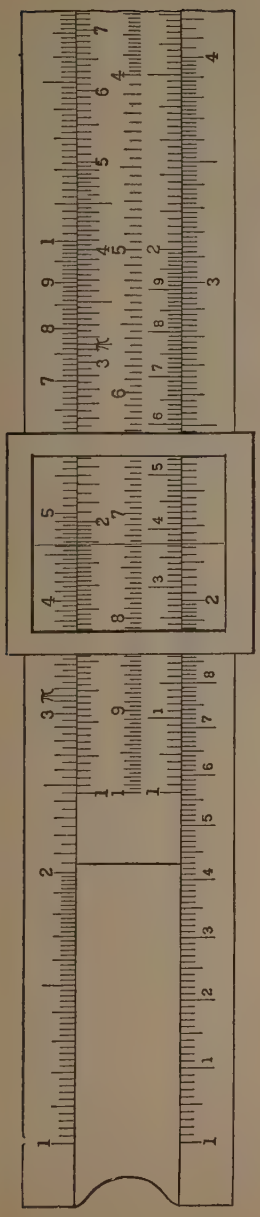


FIG. 21

decimal places, the decimal point in the result just found must be moved $n = 4$ places to the left, according to the rule of Art. **43.** When this is done, the answer is .000215; that is, $.1564 \div 728 = .000215$.

46. The *C I* scale of the rule shown in Fig. 2 may be used in division, if desired. The index of the *C* scale is set to the dividend on the *D* scale and directly under the divisor on the *C I* scale the quotient is read from the *D* scale. Let the *C I* scale be used to find the quotient of $1.564 \div 72.8$. Set the left-hand index of the *C* scale to 1564 on the *D* scale, as shown in Fig. 21; then set the hair-line over 728 on the *C I* scale, and below, where the hair-line crosses the *D* scale, will be found 215; that is, the significant figures of the result are 215. Find the position of the decimal point by approximating the original numbers; for example, 1.564 may be taken as 1.4 and 72.8 as 70. Then, $1.4 \div 70 = .02$, showing that one cipher must come between the decimal point and the first significant figure. Hence, the result is .0215.

When the *C I* scale is used, the rules given in Arts. **41** to **43** cannot be used as they stand; but if *right* is substituted for *left* and *left* for *right* in the statement of these rules, then they will apply to the use of the *C I* scale.

47. In certain classes of work it may be found necessary to determine the quotients of a number divided by a series of other numbers; that is, a constant dividend is to be divided successively by a number of divisors. This may be done by using the *C* and *D* scales, as described in Art. **37;** but the slide will have to be reset for each divisor. It is better, therefore, to use the *C I* scale, because, in many cases, after the index of the slide is set to the dividend on the *D* scale, there is no need to move the slide again, and the runner can be moved so as to indicate the several successive divisors.

Similarly, a constant may be used as a divisor in a series of calculations. In such a case, set the index of the *C* scale to the constant divisor on the *D* scale, and then read the quotient on the *C* scale above the dividend on the *D* scale. In this way, the slide will not need to be moved more than once and the hair-line can be set to the successive dividends on the *D* scale.

EXAMPLES FOR PRACTICE

Find the quotient of:

(*a*) 166 ÷ 13.
(*b*) 287 ÷ 45.
(*c*) 3,125 ÷ 125.
(*d*) 78 ÷ 16.1.
(*e*) 13.9 ÷ 6.72.
(*f*) 160 ÷ 936.
(*g*) 54.27 ÷ 25.
(*h*) 1.05 ÷ .18.
(*i*) 2,075 ÷ 8.38.
(*j*) .854 ÷ .1065.
(*k*) 3.635 ÷ 272.4.
(*l*) .12 ÷ .192.
(*m*) .0212 ÷ .706.
(*n*) 542.7 ÷ 7,236.
(*o*) 63.3 ÷ 128.75.

Ans.
(*a*) 12.77
(*b*) 6.38
(*c*) 25
(*d*) 4.84
(*e*) 2.07
(*f*) .171
(*g*) 2.17
(*h*) 5.83
(*i*) 247.5
(*j*) 8.02
(*k*) .0133
(*l*) .625
(*m*) .03
(*n*) .075
(*o*) .492

MULTIPLICATION AND DIVISION COMBINED

48. All of the examples heretofore dealt with have contained whole numbers and decimals; but in ordinary calculations it will be found that fractions are often involved. For

instance, suppose that it is required to find the result of the expression $\frac{128}{360}(.375\times 9.27\times \frac{11}{16})$. It is evident that a series of operations, involving both multiplication and division, must be performed. The first step is to simplify the expression, which then becomes $\frac{128\times .375\times 9.27\times 11}{360\times 16}$. Here, then, is a problem in which the product in the numerator is to be divided by the product in the denominator. The continued multiplication of the factors in the numerator is 4,900, according to the method of Art. **31,** and the product of 360×16 is 5,760. Then, applying the method described in Art **42,** $4{,}900\div 5{,}760 = .85$, which is the value of the original expression.

49. Another way of solving the foregoing example is to perform multiplication and division alternately. Thus, the several steps would be as follows: Multiply 128 by .375; divide that product by 360; multiply the quotient by 9.27; divide that product by 16; and multiply the last quotient by 11. The result will be .85, as before. This method is continuous; that is, it is not necessary to make two sets of multiplications and read the products before making the final division. Instead, the runner is set to mark each successive product or quotient obtained, but none of these values are read until the final product is reached.

50. The position of the decimal point in case of combined multiplication and division may be found by the rules already given; but that involves noting how often the slide projects to the right and to the left in multiplying and dividing, and considerable time is required to apply the rules. So it is better to learn the method of approximation already described. Take the example of Art. **48,** for instance. If the several numbers are roughly approximated, the problem may be set down as $\frac{100\times \frac{4}{10}\times 10\times 10}{400\times 20}$, which, when simplified, is $\frac{100\times 4\times 10\times 10}{400\times 20\times 10}$. By cancelation this expression reduces to

$\frac{1}{2} = 5$, showing that the first significant figure of the answer must directly follow the decimal point, which agrees with the result found in Art. 48.

EXAMPLES FOR PRACTICE

Find the values of the following expressions:

(*a*) $\dfrac{.012 \times 375 \times 86.4 \times 80.9}{72 \times 1{,}525}$.

(*b*) $\dfrac{.0083 \times 1{,}785 \times 824}{103} \div \dfrac{257}{26}$.

(*c*) $\dfrac{36.72}{13.9} \times \dfrac{4{,}556}{2.780} \times \dfrac{36.3}{127.85} \times \dfrac{27}{28}$.

(*d*) $1{,}675 \times \dfrac{87.3}{12} \times 126 \times \dfrac{.566}{42.83}$.

Ans. (*a*) .286; (*b*) 11.99; (*c*) 1.185; (*d*) 20,300

RECIPROCALS

51. The reciprocal of a number is the quotient obtained by dividing 1 by that number. The slide rule furnishes a very rapid means of finding the reciprocals of numbers. If the rule is of the type shown in Fig. 2, the reciprocal of any number on the *C* scale is found directly above, on the *C I* scale; or the reciprocal may be found on the *C* scale directly below the given number on the *C I* scale.

If the rule is of the type shown in Fig. 1, set the index of the *C* scale to the given number and read its reciprocal from the *C* scale opposite the index of the *D* scale; or, set the index of the *D* scale to the given number on the *C* scale and read the reciprocal from the *D* scale opposite the index of the *C* scale; or, turn the slide end for end, with the *B* scale next the *D* scale and set the indexes in line at the ends. Then the *C* scale will be upside down, and will read from 1 at the right to 10 at the left. In other words, it will be practically a *C I* scale and can be used in the same way to find the reciprocal of any number.

52. Since the values on the *C I* scale are the reciprocals of the values directly beneath on the *C* scale, it follows that

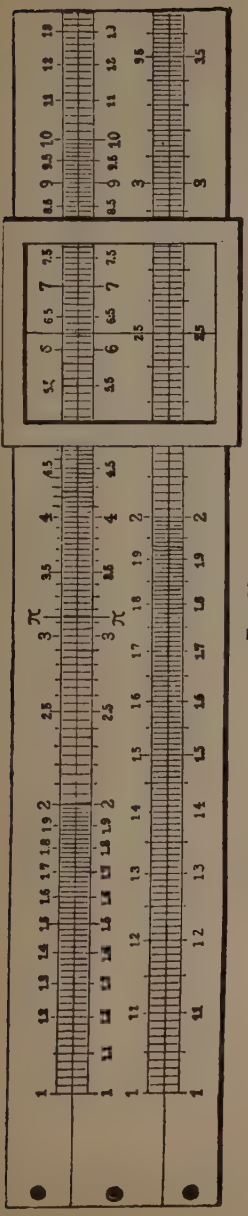


FIG. 22

multiplying by a given value on the C scale is equivalent to dividing by the corresponding reciprocal on the CI scale. Suppose, for example, that the problem is 12.6×6. The multiplier is 6 and its reciprocal is $\frac{1}{6}$. But, 12.6×6 has exactly the same value as $12.6 \div \frac{1}{6}$, for in each case the answer is 75.6.

It is advantageous to use the CI scale for multiplying and the C scale for dividing, because there is no need of determining which index of the scale must be set to one of the numbers. In division, the divisor on the C scale is set over the dividend on the D scale and the quotient read from the D scale opposite the index of the C scale; and in multiplication one factor on the CI scale is set over the other on the D scale and the product is read from the D scale opposite the index of the C scale.

POWERS

53. The slide rule furnishes a convenient means for finding the square or the cube of a number. If the square is to be found, the A and D scales are used. The runner is set so that the hair-line stands over the given number on the D scale and the square is read from the A scale where the hair-line crosses it. For example, let the square of 2.5 be required. The setting of the rule, as in Fig. 22, gives 625 as the significant figures. If 2.5 is taken as 3, approximately, the square of that value is 9, showing that the square of 2.5

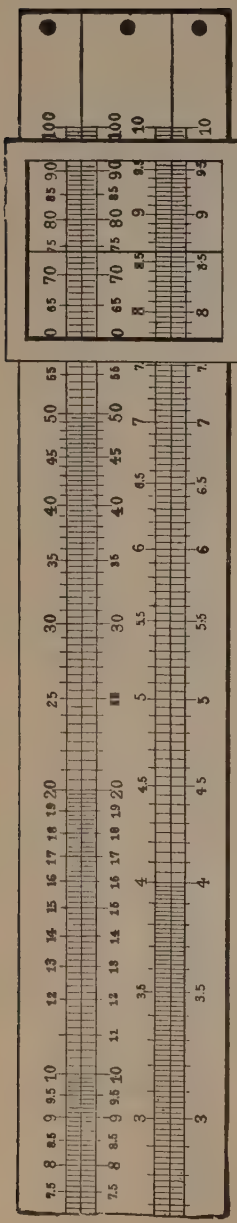


FIG. 23

must be something less than 9 and yet have one figure in the whole-number part. Therefore, the decimal point is placed after the first significant figure, and 6.25 is the required answer.

54. The position of the decimal point may be found, also, by the following rule:

Rule.—*If the square is read from the left-hand A scale, the number of figures in the answer is 1 less than twice the number of figures in the given number; if it is read from the right-hand A scale, the number of figures in the answer is twice the number of figures in the given number. In either case, point off twice as many decimal places as there are in the given number.*

In the preceding example, the given number contains two figures and one decimal place and the square is read from the left-hand *A* scale. Therefore, according to the rule, the number of figures in the result must be $(2\times2)-1=3$, and the number of decimal places to be pointed off is $2\times1=2$; hence, 6.25 is the correct result.

55. Let it be required to find the square of 8.6. Set the runner at 86 on the *D* scale, as shown in Fig. 23, and at the point where the hair-line crosses the *A* scale the reading is 74. As the answer is read from the right-hand *A* scale, there must be $2\times2=4$ significant figures in the result; hence,

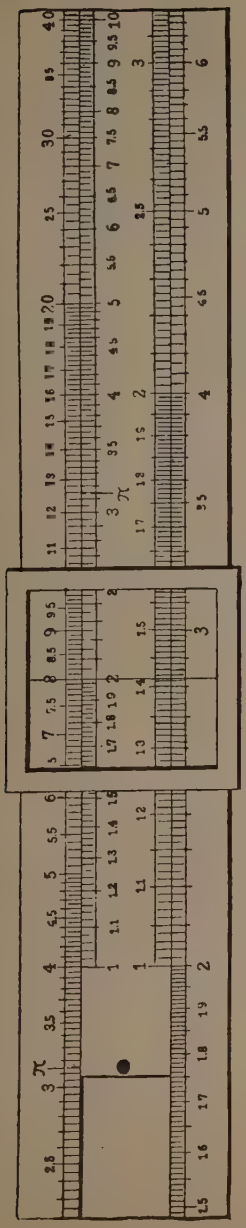


FIG. 24

ciphers are annexed, and the number becomes 7,400. But, as the original number contains 1 decimal place, the square must have $2 \times 1 = 2$ decimal places. If two decimal places are pointed off in 7,400, the result is 74.00, or simply 74, which is the square of 8.6, according to the slide rule. The exact square of 8.6 is 73.96.

Suppose that the square of .027 is required. Set the runner to 27 on the *D* scale and on the *A* scale directly above read 73; then the first two significant figures of the answer are 73. As the reading is on the left-hand *A* scale, the number of significant figures in the result is 1 less than twice the number of significant figures in the given number, or $(2 \times 2) - 1 = 3$; hence, a cipher is annexed to give the three figures necessary, and the result becomes 730. But, it is necessary to point off twice as many decimal places as in the original number, or $2 \times 3 = 6$; when 6 decimal places are pointed off in 730, the answer is .000730, or simply .00073. The exact square of .027 is .000729.

56. In finding the cube of a number with the rule shown in Fig. 1, it is necessary to use the four scales *A*, *B*, *C*, and *D*, as well as the runner. The index of the *C* scale is set to the given number on the *D* scale. Then the cube of the number is read from the *A* scale directly over the given number on the *left-hand B* scale.

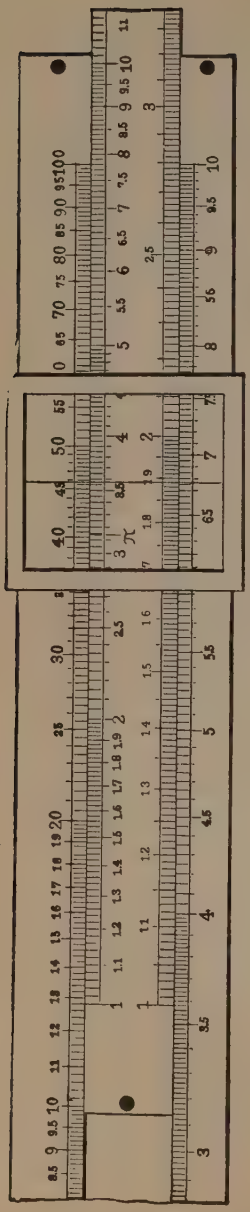


FIG. 25

Suppose, for example, that the cube of 2 is desired. Place the left-hand index of the *C* scale opposite 2 on the *D* scale. Then, on the *A* scale, opposite 2 on the *B* scale, is 8, the required answer. The setting of the slide and runner is clearly shown in Fig. 24.

57. If the *C* and *D* scales are set so that their indexes coincide, and the *C* scale is then slowly moved to the right, it will be found that, according to the statements in the preceding article, the cubes of all numbers from 1 to 2.154 can be read from the left-hand *A* scale. The cubes of numbers from 2.154 to 4.641 are read from the right-hand *A* scale. In order to obtain the cubes of numbers between 4.641 and 10, it is necessary to move the slide to the left, so as to bring the right-hand index of the *C* scale opposite the given number on the *D* scale, and to read the result from the *A* scale.

58. The three cases that may arise in finding the cube of a number by the slide rule may be summed up as follows:

1. Slide moved to the right and the cube read from the left-hand *A* scale.

2. Slide moved to the right and the cube read from the right-hand *A* scale.

3. Slide moved to the left and the cube read from the left-hand *A* scale.

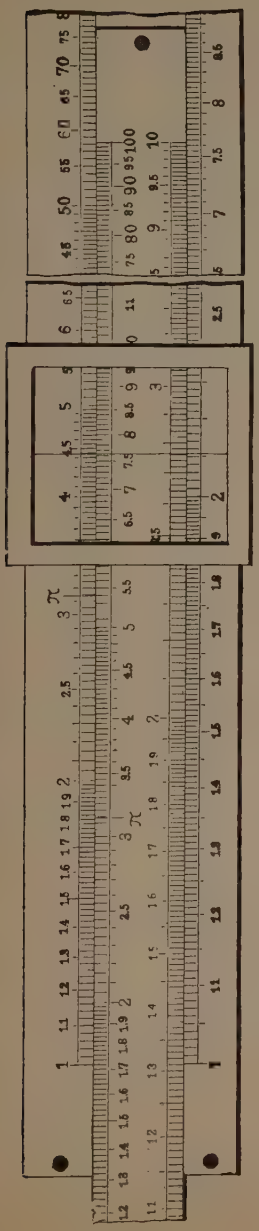


Fig. 26

Let n represent the number of figures in the given number and N the number of figures in the cube. Then the position of the decimal point may be obtained by observing the following rule:

Rule.—*If the result is obtained with the setting described in Case 1, then* $N=3n-2$; *if by that in Case 2,* $N=3n-1$; *if by that in Case 3,* $N=3n$. *The number of decimal places is always three times the number of decimal places in the given number.*

59. Let the cube of 3.58 be required. The left-hand index of the C scale is set to 358 on the D scale, as in Fig. 25. Then, opposite 358 on the left-hand B scale, the first three significant figures of the result are found from the right-hand A scale to be 458. The cube is found by the setting described in Case 2, Art. **58,** and the value of n is 3. Then, $N=3\times3-1=8$, and there are $3\times2=6$ decimal places. Hence $3.58^3=45.800000$, or 45.8.

Again, suppose that 76.25 is to be cubed. The right-hand index of the C scale is set to 7625 on the D scale, as in Fig. 26. Then, on the left-hand A scale, opposite 7625 on the left-hand B scale, is found 444; that is, the first three significant figures of the answer are 444. The cube was found by the setting described in Case 3, Art. **58,** and the value of n is 4. Then, $N=3\times4=12$, and there are

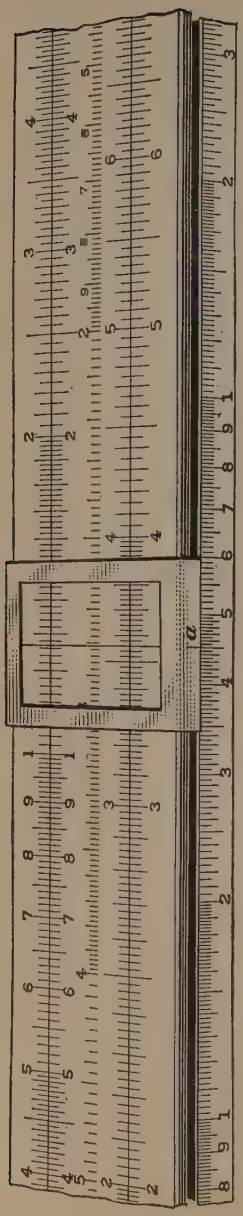


FIG. 27

$3\times2=6$ decimal places. Therefore, $76.25^3=444{,}000.000{,}000$, or 444,000.

60. The cube of a number may be found very quickly by using the *E* scale on the edge of the rule of the type shown in Fig. 27. The edge of the runner is scribed with a line *a* that is exactly in line with the hair-line on the glass. Therefore, to find the cube of a number, set the hair-line to this number on the *D* scale, and in line with the mark *a* on the edge of the runner, read the cube from the *E* scale. The *E* scale has three parts that are alike. If the cube is found on the left-hand *E* scale, it contains 2 less than three times the number of figures in the original number; if on the middle *E* scale, 1 less than three times the number of figures in the original number; and if on the right-hand *E* scale, three times as many figures as the original number. The number of decimal places in the answer is three times the number of decimal places in the original number.

61. Suppose that the cube of 35.6 is to be found by the rule shown in Fig. 2. Set the runner to 356 on the *D* scale, as in Fig. 27, and below the mark *a* on the edge of the runner will be found 451 on the middle *E* scale. The number of significant figures in the result, according to the preceding article, is 1 less than three times the number of significant figures in the original number, or $(3\times3)-1=8$.

Hence, the result is 45100000. But, the number of decimal places in the original number is 1, and so there will be $3 \times 1 = 3$ decimal places in the answer; hence, 45100.000, or 45,100, is the cube of 35.6.

62. It is possible to find higher powers of numbers by using the slide rule. Thus, the value of 2.3^5 might be required. As $2.3^5 = 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3$, the problem becomes simply a case of continued multiplication and is solved by the rules already given for problems of that character. But, as $2.3^5 = 2.3^2 \times 2.3^2 \times 2.3 = (2.3^2)^2 \times 2.3$, a quicker method of solving is to square 2.3, then square that product and multiply the result by 2.3. The number of decimal places to be pointed off in the result is five times the number of decimal places in the given number. The fourth power may be found by squaring the square of the number and the sixth power by squaring the cube of the number.

ROOTS

63. The process of finding the root of a number is the reverse of raising a number to a given power. Thus, to find the square root of a number, set the hair-line of the runner over the given number on the *A* scale, and directly below, on the *D* scale, read the square root. If the given number has an odd number of figures preceding the decimal point, or an odd number of ciphers directly following the decimal point in case the given number is *wholly* a decimal, it is necessary to use the left-hand *A* scale. The number of figures preceding the decimal point in the root is found by the formula $M = \frac{m+1}{2}$, in which M denotes the number of figures preceding the decimal point in the root and m denotes the number of figures preceding the decimal point in the given number. The number of ciphers directly following the decimal point in the root, in case the given number is wholly a decimal is found by the formula $M = \frac{m-1}{2}$, in which M is the number of ciphers directly following the decimal point in the root and m is the number of ciphers directly following the decimal

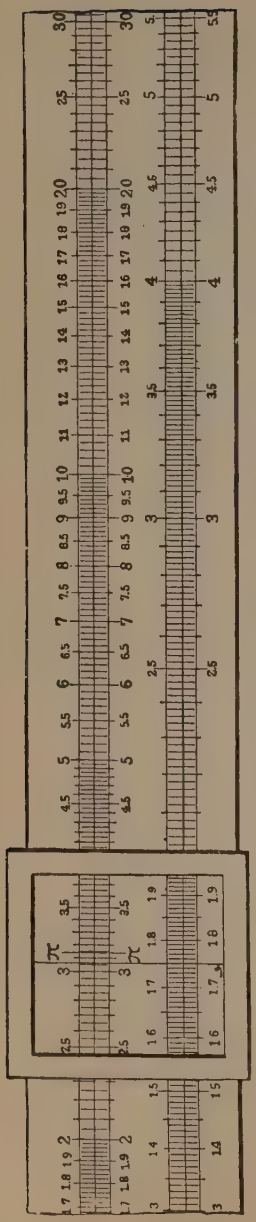


FIG. 28

point in the given number. These formulas are used only in those cases in which m is an *odd* number.

64. If the given number has an *even* number of figures preceding the decimal point, or an even number of ciphers directly following it, the right-hand A scale must be used, and the formula then becomes $M = \frac{m}{2}$.

Let it be required to find the square root of 306.25. As there is an odd number of figures preceding the decimal point, the left-hand A scale is used, and the runner is set to 306, as shown in Fig. 28. The corresponding reading on the D scale, to three significant figures, is 175. According to to the formula, $M = \frac{3+1}{2} = 2$; that is, two figures precede the decimal point in the root. Hence, the result is 17.5.

65. The method of finding the number of figures in the cube root is to point off the number into periods of three figures each, counting in each direction from the decimal point. The root will then contain one figure for each period or fraction of a period thus set off. If the number is a decimal or a mixed number, ciphers must be annexed at the right until the number of figures to the right of the decimal point is exactly divisible by 3.

If the first period contains one digit, the left-hand index of the C scale is used in connection with the

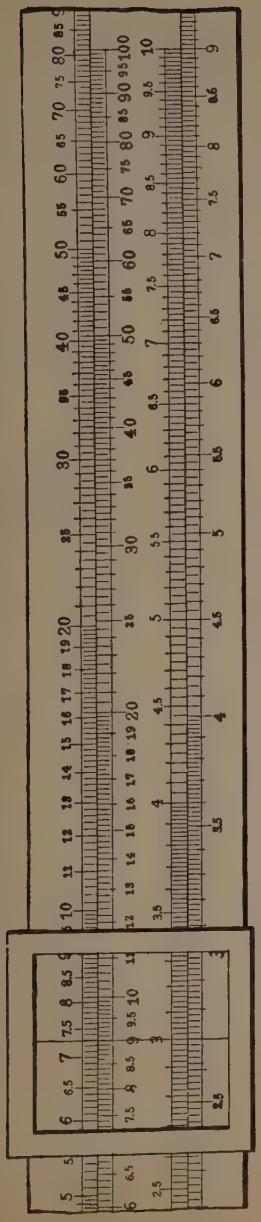


FIG. 29

left-hand A scale. If the first period contains two digits, the left-hand index of the C scale is used in connection with the right-hand A scale. If the first period contains three digits, the right-hand index of the C scale is used in connection with the left-hand A scale.

66. In order to extract the cube root of a number, the process of cubing is reversed. First, the runner is set to the reading on the scale A corresponding to the number whose cube root is desired. Then the slide is moved to the right or to the left until the reading on the B scale under the hair-line is the same as that on the D scale opposite the index of the C scale. This latter reading gives the significant figures of the answer. Thus, let $\sqrt[3]{729}$ be required. The runner is first set to 729 on the left-hand A scale. Then the slide is moved to the left until a point is reached at which the reading of the B scale under the hair-line is the same as the reading on the D scale opposite the right-hand index of the C scale. This reading is found to be 9, as shown in Fig. 29. Hence, $\sqrt[3]{729} = 9$.

67. Suppose that the cube root of 18,225 is to be found. On pointing off, the number is found to contain two periods, and so the root has two figures preceding the decimal point. The first period contains two digits, and so the left-hand index of the

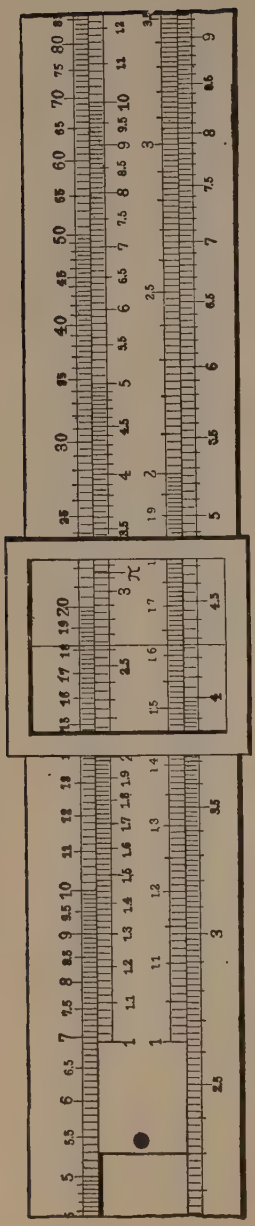


FIG. 30

C scale is used with the right-hand A scale. The runner is first set to 182 on the right-hand A scale, as shown in Fig. 30, and the slide is then moved to the right until the reading on the B scale under the hair-line is the same as that on the D scale opposite the left-hand index of the C scale. This reading is found to be 263, to three significant figures. As there must be two figures to the left of the decimal point, the correct answer is 26.3.

68. Again, let the cube root of .00256 be required. A cipher is added, so as to produce two complete periods of three figures each, or .002′560. The first period has but one digit, 2, and so it is necessary to use the left-hand index of the C scale with the left-hand A scale. The runner is set to 256 on the left-hand A scale and the slide is moved to the right until the reading on the B scale under the hair-line is the same as that on the D scale opposite the left-hand index of the C scale. This reading is 137. As the original number is wholly decimal, and the first period contains a digit, the first significant figure of the root directly follows the decimal point; hence, the required root is .137, to three significant figures.

69. The E scale on the edge of the rule shown in Fig. 2 may be used for finding cube root. The given

number is first set off into periods of three figures each, as in the preceding instructions. If the first period contains one significant figure, the left-hand E scale is used; if it contains two significant figures, the middle E scale is used; and if it contains three significant figures, the right-hand E scale is used. The rules for finding the number of significant figures and the decimal point in the root are the same as those already given.

For example, let the cube root of 45.5 be required. Ciphers are annexed to make the decimal part a complete period, and the number becomes 45.500. As there are two significant figures in the first period, the middle E scale is used. The mark on the runner is set to 455 on the middle E scale, as shown in Fig. 27, and the root is read from the D scale under the hair-line, where it is found to be 356. As there is one period preceding the decimal point in the given number, one figure will precede the decimal point in the answer, and 3.56 is therefore the cube root of 45.5.

70. As the fourth power of any number is equal to the square of the square of that number, it is also true that the fourth root of a number is equal to the square root of the square root of the number. That is, to find the fourth root of a given number, extract the square root, and then extract the square root of that result. The method of using the slide rule for extracting square root has previously been described.

EXAMPLES FOR PRACTICE

Find the result of:

(*a*)	26^2.	Ans.	(*a*) 676
(*b*)	129^3.		(*b*) 2,150,000
(*c*)	85^4.		(*c*) 52,200,000
(*d*)	56^5.		(*d*) 551,000,000
(*e*)	4.72^3.		(*e*) 105
(*f*)	19.65^2.		(*f*) 386
(*g*)	$.0894^3$.		(*g*) .000715
(*h*)	$\sqrt{498}$.		(*h*) 22.3
(*i*)	$\sqrt[3]{6,125}$.		(*i*) 18.3
(*j*)	$\sqrt[4]{127.44}$.		(*j*) 3.36
(*k*)	$\sqrt{.04365}$.		(*k*) .209

MENSURATION

71. Area of Parallelogram.—The area a of a parallelogram is equal to the product of the base b and the altitude c; or, $a = b\,c$. Therefore, if the slide rule is used, the problem is simply one of multiplication. Set the index of the C scale to the value of b on the D scale and opposite the value of c on the C scale read the area a on the D scale. For example, let it be required to find the area of a rectangle 8.8 feet wide and 37.5 feet long. Set the index of the C scale to 88 on the D scale and opposite 375 on the C scale will be found 33 on the D scale. The slide extends to the left, as may be seen from Fig. 8, which shows this setting; therefore, the total number of figures in the product of 88 and 375 is $2+3=5$. There are two decimal places to be pointed off, and so the answer is 330.00, or 330 square feet.

72. Area of Triangle.—The area a of a triangle is equal to half the product of the base b and the altitude c, or, $a = \frac{1}{2}\,b\,c = \frac{b\,c}{2}$. In finding such an area by the slide rule, then, the method would be to multiply the values of b and c together and divide the product by 2. Suppose that a triangle has a base of 74.2 feet and an altitude of 29.7 feet, and the area is to be found. Set the right-hand index of the C scale to 742 on the D scale, and move the runner over 297 on the C scale. Then move the slide so as to bring 2 on the C scale under the hair-line, and opposite the left-hand index of the C scale read 11 on the D scale. Following the usual rules, the answer is found to be 1,100 square feet.

73. Circumference of Circle.—The circumference c of a circle is 3.1416 times the diameter d, or, $c = \pi\,d$, the character π signifying 3.1416. Therefore, to find the circumference of a circle, proceed as in ordinary multiplication. To make this sort of calculation easy, the value corresponding to 3.1416 is marked on the left-hand A and B scales, and is indicated by π. Thus, if the circumference of a 21-inch circle is to be found, the left-hand index of the B scale is set

to π on the A scale and opposite 21 on the B scale is found 66 on the A scale. Placing the decimal point by inspection, the answer is 66 inches.

74. Area of Circle.—The area of a circle is equal to .7854 times the square of the diameter. On some makes of slide rules, there is an extra line on the right-hand A and B scales to designate the value 7854. Hence, to find the area of a circle 24 inches in diameter, for example, set 7854 on the B scale to the right-hand or left-hand index of the A scale, and above 24 on the D scale read 452 on the B scale. Inspection of the original numbers shows that the result must have three figures preceding the decimal point; therefore, the area is 452 square inches. In other words, to find the area of a circle, set 7854 on the B scale to one of the indexes of the A scale, and read the area from the B scale directly above the given diameter on the D scale.

75. Another way of obtaining the same result is to set the right-hand or left-hand index of the B scale directly over the given diameter on the D scale, and then above 7854 on the B scale read the area on the A scale. For example, let the area of a 24-inch circle be required. Set the right-hand or left-hand index of the B scale to 24 on the D scale, and above 7854 on the B scale will be found 452 on the A scale; hence, the required area is 452 square inches.

If the area is given and the diameter is to be found, the setting is exactly the same as in either of the two methods just given, but is made in the reverse manner, and the final reading is taken from the D scale opposite the index of the C scale.

PROPORTION

76. If three terms of a proportion are known, the fourth may readily be found by the slide rule. Locate the first term on the A scale, set beneath it the second term on the B scale, and below the third term on the A scale read the fourth term on the B scale. For example, suppose that the value of x is to be found in the proportion $26 : 78 = 5 : x$. Beneath 26 on

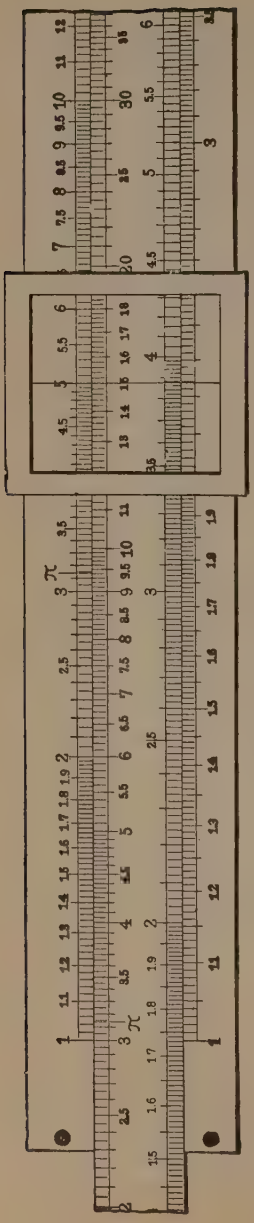


FIG. 31

the A scale set 78 on the B scale, as in Fig. 31, and under 5 on the A scale will be found 15 on the B scale. From inspection of the original proportion, it will be seen that the true answer is 15.

In case the first two terms have significant figures that are nearly the same, it will be found more convenient to use the C and D scales, as more accurate results can be obtained. In such a case, substitute C for B and D for A in the foregoing explanation.

TRIGONOMETRIC FUNCTIONS*

77. The slide rule may be used for finding the various trigonometric functions. On the back of the slide are three scales, marked S, L, and T, and known respectively as the sine scale, the logarithm scale, and the tangent scale. The scale S is graduated to designate angles between 34′ 23″ and 90°. The graduations of the T scale are for angles between 5° 43′ and 45°. The L scale is a scale of equal parts and is used in finding the logarithms of numbers.

If the slide rule is turned over, a cut-out will be found at one end, as shown in Fig. 32, and across this open-

*Arts. **77** to **85**, inclusive, deal with the use of the slide rule for finding trigonometric functions of angles and logarithms of numbers. They may be disregarded by those not familiar with these subjects. The slide rule is seldom used for these purposes, as the required values may be taken from tables.

ing is a piece of transparent celluloid on which is marked a hair-line *a*. When the *S* and *T* scales are used, the slide is drawn out until the given angle on the scale comes under the hair-line.

In some makes of rules, there is a cut-out at each end and each bears a mark corresponding to a hair-line.

78. Finding the Sine.—To find the sine of any angle between 34′ 23″ and 90°, draw out the slide until the given angle on the *S* scale is beneath the hair-line on the cut-out at the back of the rule. Then, without disturbing the setting, turn the rule over and read the sine from the *B* scale opposite the right-hand index of the *A* scale. As an example, let the sine of 8° 20′ be required. Set 8° 20′ on the *S* scale under the mark *a*, as shown in Fig. 32, turn the rule over, and oppo-

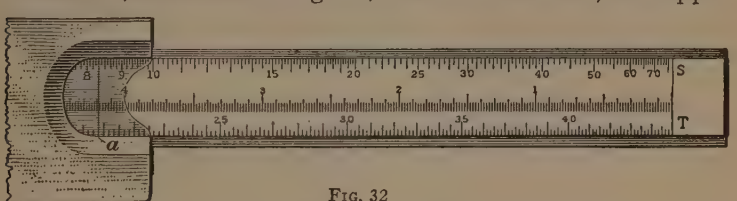


FIG. 32

site the index of the *A* scale read 145 on the *B* scale as the significant figures of the sine. The position of the decimal point is determined as follows:

Rule.—*If the result is found on the right-hand B scale, the first significant figure directly follows the decimal point; if on the left-hand B scale, a cipher intervenes between the decimal point and the first significant figure.*

In the foregoing example, the result is read from the right-hand *B* scale, and so .145 is the value of the sine required.

79. As the sine of 90° is 1, the right-hand index of the *B* scale represents 1 when that scale is used for finding the values of sines; therefore, the value of the middle index of the *B* scale is .1 and of the left-hand index is .01. As .01 is the value of the sine of 34′ 23″, that is the smallest angle whose sine can be found by the direct use of the scale *B*.

80. Finding the Cosine.—According to trigonometry, the cosine of any angle is equal to the sine of 90° minus that

angle; hence, if the cosine of an angle is to be found, subtract the angle from 90° and find the sine of the remainder, as already explained. The result will be the required cosine. For example, let cos 58° 30′ be required. As cos 58° 30′ $=\sin(90°-58°\ 30')=\sin 31°\ 30'$, set the S scale with 31° 30′ under the mark a, Fig. 32, turn the rule over, and read 523 as the significant figures of the result. This reading is found on the right-hand B scale, opposite the index of the A scale, and so, by the rule of Art. **78**, the decimal point directly precedes the first figure; hence, cos 58° 30′ = .523.

81. Finding the Tangent.—To find the tangent of an angle *less* than 45° set the given angle on the T scale under the mark a, Fig. 32, turn the rule over, and read the value of the tangent from the C scale opposite the index of the D scale. As $\tan 45° = 1$, the right-hand index of the C scale has a value of 1 for finding tangents; then the left-hand index has a value of .1, which is the tangent of 5° 43′, the smallest angle on the T scale. If the angle is greater than 45°, it is first subtracted from 90°, the remainder on the T scale is set under the mark a, and the desired tangent is read from the D scale opposite the index of the C scale. If the angle is less than 45°, the first significant figure of the result follows the decimal point directly; if it is greater than 45°, the decimal point follows the first figure.

82. Let the tangent of 17° 25′ be required. Set the T scale so that 17° 25′ is under the hair-line at the back, turn the rule over, and read 314 as the significant figures of the result, found on the C scale opposite the index of the D scale. As the angle is less than 45° the decimal point precedes the first figure; that is, tan 17° 25′ = .314.

Suppose that the given angle is 72° 35′. This is greater than 45°, and so it is subtracted from 90° and the remainder is used in making the setting; that is, $90°-72°\ 35'=17°\ 25'$, which is the value on the T scale to be set to the hair-line. The significant figures 319 are read from the D scale opposite the index of the C scale. As the given angle exceeds 45°, the decimal point follows the first figure, and so tan 72° 35′ = 3.19.

As 5° 43′ is the smallest angle on the T scale, $90° - 5° 43' = 84° 17'$ is the largest angle whose tangent can be found from the scale in the manner described.

83. Finding the Cotangent.—According to trigonometry, the cotangent of any angle has the same value as the tangent of 90° minus that angle. If the cotangent of any angle between 5° 43′ and 84° 17′ is to be found by the slide rule, subtract the given angle from 90° and find the tangent of the remainder, in the manner just described. The result will be the cotangent of the original angle.

84. Finding the Secant or Cosecant.—The secant of an angle is the reciprocal of the cosine of the angle, and the cosecant is the reciprocal of the sine of the angle. A simple way to find the secant, therefore, is to find the cosine of the given angle and then take the reciprocal of that value. Similarly, to find the cosecant, find the value of the sine of the given angle and then take the reciprocal of that value.

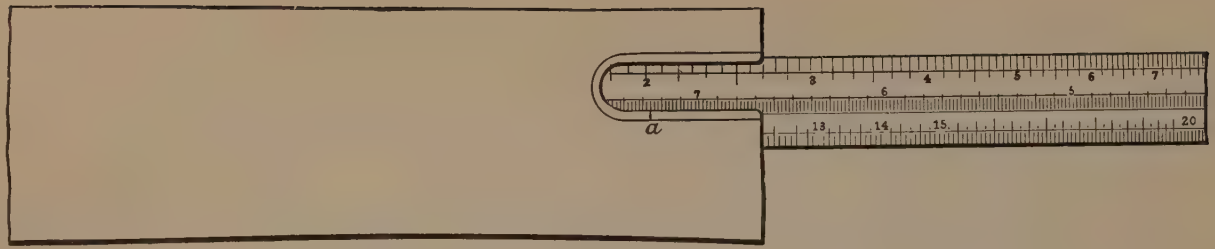
EXAMPLES FOR PRACTICE

Find the value of:

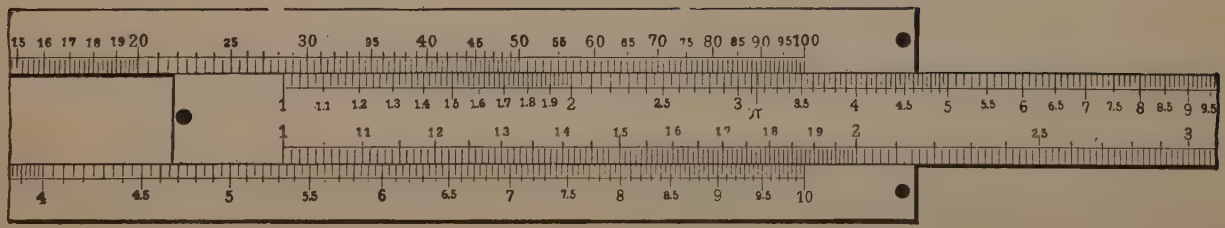
		Ans.	
(*a*)	sin 16° 30′.		(*a*) .284
(*b*)	cos 38° 15′		(*b*) .785
(*c*)	tan 20° 50′.		(*c*) .381
(*d*)	cot 7° 35′.		(*d*) 7.51
(*e*)	sec 32° 45′.		(*e*) 1.19
(*f*)	cosec 10° 30′.		(*f*) 5.49
(*g*)	tan 78° 15′.		(*g*) 4.81
(*h*)	cos 66° 30′.		(*h*) .399
(*i*)	cot 80° 45′.		(*i*) .163
(*j*)	sin 57° 30′.		(*j*) .843
(*k*)	sin 49° 15′.		(*k*) .758
(*l*)	cos 52° 45′.		(*l*) .605

LOGARITHMS

85. The mantissa of the logarithm of any number may be found to three significant figures on the slide rule. On the under side of the slide, between the S and T scales, is the



(*a*)



(*b*)

FIG. 33

logarithmic scale, which will be called the *L* scale. To find the mantissa of the logarithm of a number, move the slide to the right until the left-hand index of the *C* scale is opposite the given number on the *D* scale, and then, without disturbing the position of the slide, turn the entire rule over and read the mantissa from the *L* scale under the index line on the back, as shown at *a*, Fig. 33 (*a*). The characteristic is found in the usual manner. It must be remembered that the right-hand index of the *L* scale denotes zero and the left-hand index denotes 1. If this fact is kept in mind, there will be no difficulty in finding the number of ciphers between the decimal point and the first significant figure of the mantissa. If the first three significant figures of the given number are 126, or some greater number than this, the decimal point in the mantissa directly precedes the first significant figure.

Suppose that the logarithm of 5,325 is required. Set the left-hand index of the *C* scale to 5325 on the *D* scale, as in Fig. 33 (*b*), and on the *L* scale, as in (*a*), read 726. The characteristic is 3; hence, the logarithm is 3.726.

GENERAL REMARKS

86. It is not intended that the foregoing instructions shall form a complete treatise on the uses of the slide rule. There are very many settings that may be made for solving a variety of examples. No attempt has been made to describe these special settings, because they refer to the less usual forms of problems. If any one has a large number of such calculations to make, he may readily find the special settings required by referring to the book of instructions ordinarily supplied by the maker of the slide rule he is using. The aim of the foregoing articles is to make clear the methods of performing ordinary calculations such as may be met with in every-day work.

The various rules for finding the number of figures and the position of the decimal point in the answer are necessary for the guidance of the person who knows little or nothing about the use of the slide rule; but after the user of a rule has become

familiar with the various methods to be followed, he will find that the determination of the number of figures and the position of the decimal point will become a matter of habit, and that he will not need to pause to recall the particular rule or rules to be applied. In fact, if the slide rule is to become a saver of time in making calculations, the user of it must become so familiar with it, by continual practice, that he will not need to look up any rules; and, as has been stated before, this familiarity comes only by long-continued use of the slide rule.

87. One of the principal points to be observed in using the slide rule is to set the slide and the runner accurately. The lines that indicate the given numbers, if on adjacent scales, should be set so as to match exactly and form one straight line. Similarly, the hair-line on the runner should be set so that it indicates correctly the desired number on any scale.

When using the runner, the light should come from above or from a point directly in front of the operator. If the source of light is at either side, a shadow will be cast on the scales by the hair-line, and the shadow will confuse the reading. The runner should be examined to see that it is accurately set in its grooves. To do so, set the hair-line over 2, 3, 4, etc. on the *D* scale and see whether it crosses 4, 9, 16, etc. on the *A* scale. If it does not do this exactly, it is either faultily made or incorrectly adjusted, and it will not give accurate results.

88. The principle on which all slide rules are based is the same, although the rules may differ greatly in the arrangement, number, and use of the scales. The preceding paragraphs describe rules such as are in common use for ordinary calculations, but there are a great variety of rules made for special purposes. For example, there are rules with special graduations, and sometimes with more than one slide, for calculating the horsepower of engines, finding the flow of steam or water in pipes, determining discount and compound interest, determining the size of wire for electrical work, esti-

mating the sizes of sewers, and for many other purposes. Furthermore, calculating devices based on the principle of the slide rule may be made cylindrical or circular. The catalog of any maker of such devices will show the various forms and point out their special uses.

DETAILS OF CONSTRUCTION

ADJUSTMENT OF SLIDES

89. The slide should have an easy fit in its grooves, so that it may be moved with little effort and set accurately without difficulty; in other words, it should not be so loose that it will fall out if the rule is held vertical, nor should it be so tightly held that great force is required to move it.

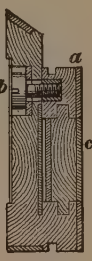


FIG. 34

Differences in weather conditions may so affect slide rules as to cause the slides to bind or to become loose. As a result, makers of rules employ various forms of construction in an endeavor to obtain smooth and easy movement of the slide in all positions and in all climates.

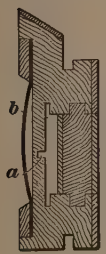


FIG. 35

90. One method of providing for the tightness of the fit of the slide is illustrated in Fig. 34. The upper strip *a* of the rule, on which the *A* scale is fixed, is made separate from the body and is held in place by three screws inserted from the back of the rule. These screws, one of which is shown at *b*, are spaced equally throughout the length of the rule. They pass through slots in the body of the rule and screw into the strip *a*. The slots are at right angles to the direction of movement of the slide, and so the strip *a* may be set over against the slide *c* with the required pressure and then firmly held by tightening the screws *b*.

91. Another form of construction intended to serve the same purpose is shown in Fig. 35. The back of the body of

the rule has a lengthwise groove *a*, and across the back of the rule, extending from end to end, is a steel spring, shown in section at *b*. The spring is firmly fastened to the back of the rule and furnishes the pressure by which the slide is held between the sides of the rule.

In yet another make of rule, the slot, or guide, in which the tongue on the upper edge of the slide moves, contains a spring made of metal or a thin strip of wood. This spring presses against the edge of the tongue and gives the friction necessary to hold the slide in any position to which it may be set.

RUNNERS

92. The metal-frame runner has the serious disadvantage that in certain positions its sides hide the figures on the scales and make it difficult to read the values beneath the hair-line. To overcome this disadvantage, the frameless type shown in Fig. 36 was constructed. It consists of an oblong piece of glass *a*, flat on both sides, held to the guides *b* and *c* by screws *d*. The guide *b* carries a steel spring *e* that bears against the upper edge of the rule and so holds the lower guide *c* flat against the lower edge of the rule. The runner is thus kept in such a position that the hair-line is always at right angles to the scales. As there are no side pieces to the runner, all the figures on the rule are visible at all times.

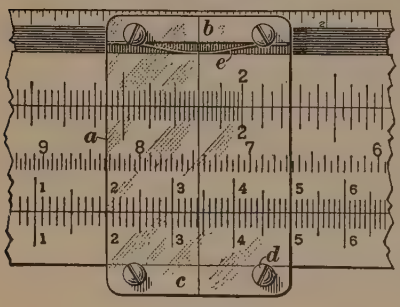


FIG. 36

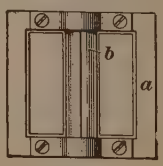


FIG. 37

93. The plain form of runner shown in preceding illustrations has a hair-line scribed on a thin square of glass. It may be found difficult to set this line accurately on parts of the scale where the divisions are small. To obviate this trouble, the magnifying

runner was designed, one form of which is shown in Fig. 37. The metal frame *a* carries a narrow strip of glass *b* that is flat on the under side, next the face of the rule, and convex upwards. The hair-line is marked on the under flat surface and the shape of the strip causes the hair-line to be magnified, as well as the divisions of the scale directly under it, and thus the setting is made easier.

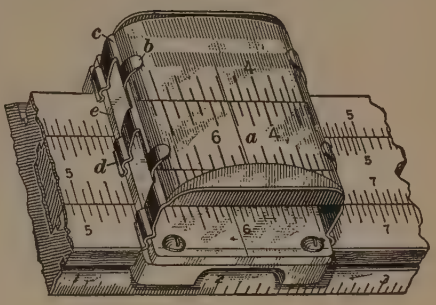


FIG. 38

94. The form of runner shown in Fig. 36 may be fitted with a magnifying device, if desired. A magnifier attached to a frameless runner is shown in Fig. 38. It consists of a convex piece of glass *a*, flat on its under side, and held by tongues *b* in a metal frame *c*. This frame has a number of clips *d* by which it may be sprung into place on the edges of the runner *e*, as shown. The glass *a* is thus held at a fixed distance above the runner and so acts as a magnifying lens.

MECHANICS AND MACHINE ELEMENTS

Serial 1621 Edition 1

PRINCIPLES OF MECHANICS

MATTER AND ENERGY

DEFINITIONS

1. By means of the five senses—*sight*, *hearing*, *smell*, *taste*, and *touch*—we learn of the existence of the world of things about us and of the appearance and the qualities of those things. The word *phenomenon* (plural, *phenomena*) means any appearance observed by these senses, or the way in which things appear to us. Thus, the phenomenon of sound is observed in the neighborhood of a waterfall; the phenomena of color and form are observed in a rainbow. The appearance of a thing, or the way it affects our senses, is the phenomena associated with it. All the phenomena that we observe can be grouped under two general heads, *matter* and *energy*.

2. **Matter** is anything that appears to occupy space. Matter of a particular kind is called a *substance*, and a definite part of a substance is called a *body*. Thus, air, water, iron, and wood are some of the forms of matter because they occupy space; each is a substance because it is a particular kind of matter; a roomful of air, a drop of water, a building, and a world are bodies because each is a definite quantity of a substance. The *volume* of a body is the extent of the space it occupies.

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3. Energy is the name given to the cause of all material activity; it is capacity for doing work. Energy may be manifested in different forms, as heat, light, and mechanical motion. Energy is said to be *kinetic* when actually producing material activity, or doing work; it is called *latent*, or *potential*, when presumably existing without any manifestation of its existence. Thus, the heat in the steam confined in a boiler is a form of latent, or potential, energy; when this heat is used in the cylinders of a steam engine, it becomes kinetic energy in the form of mechanical motion.

4. The **forms** in which matter and energy appear to exist may be changed, but the total quantity of each remains fixed. When a building burns, the substances are changed into gases, ashes, cinders, etc., but the total quantity of matter remains unchanged. In an electric-power plant, coal is burned under the boilers, and heat is stored in the steam. Some of this heat energy is converted into energy of mechanical motion in the engines, some of the mechanical motion into electric energy in the dynamos, and some of the electric energy into heat energy in the electric lamps, making them glow with light. The part of the energy not converted into the desired form at each change is often called *energy loss*, but no energy is really lost.

5. Physics, or *physical science*, is a general name given the group of sciences that treats of the phenomena associated with matter, especially in its relations to energy. There are many branches of physical science, among which are *mechanics*, *electricity*, and *chemistry*. A thorough understanding of electricity requires some knowledge of other physical sicences.

6. Mechanics is that branch of physical science which treats of the phenomena caused by the action of forces on material bodies.

CONSTITUTION AND STATES OF MATTER

7. Constitution of Matter.—The sciences treating of matter are in most cases based on the *atomic theory*. Although discoveries made in the 20th century indicate that this theory

is not wholly correct, terms, definitions, and explanations based on it are still used because no substitutes are yet available. A general understanding of some of these terms is therefore essential in order to understand electrical science.

8. The **atomic theory** is that all matter consists of minute, indivisible particles called **atoms** and that two or more atoms, usually differing in nature, unite according to a definite law to form a larger body called a **molecule.** Molecules and atoms are too small to be perceived by the senses, even with the most powerful magnifying glasses; these minute particles of matter and their combinations are treated in the *science of chemistry.*

Water consists of molecules, each containing 2 atoms of hydrogen (*H*) and 1 atom of oxygen (*O*), both of which are gases at ordinary temperatures and pressures. The chemical formula for water is therefore H_2O. Likewise, the chemical formula for sulphuric acid is H_2SO_4, meaning that each molecule contains 2 atoms of hydrogen, 1 atom of sulphur (*S*), and 4 atoms of oxygen.

9. A **chemical element** is a substance that cannot be decomposed or dissociated into other forms of matter; thus, no chemical experiments with hydrogen gas reveal atoms of any other substance in it. Approximately eighty chemical elements are now listed, among them being hydrogen, oxygen, nitrogen, sulphur, carbon, iron, gold, and other metals. Coal and wood are composed very largely of carbon. Air is a mixture chiefly of oxygen and nitrogen gases. The burning of coal and wood is the chemical combination of carbon atoms and oxygen atoms; the rusting of iron is the chemical combination of iron atoms and oxygen atoms; both may be called *oxidation processes*, the first being more rapid than the second.

10. States of Matter.—Matter exists in three states, *solid*, *liquid*, and *gaseous*. In the solid state, bodies preserve constant shape and volume; in the liquid state, bodies readily assume the shape of the containing vessel but preserve constant volume; in the gaseous state, bodies expand indefinitely, preserving neither constant shape nor constant volume.

Nearly all substances can be made to exist in any one of the three forms by changing the degree of heat and pressure. Thus, by applying heat, ice can be changed to water and then to steam, and by withdrawing heat steam can be changed to water and then to ice. By producing extreme cold (withdrawing heat) and applying heavy pressure, air, which is a mixture of gases, can be liquefied and then frozen.

11. A **permanent gas** is a substance that remains in a gaseous state at ordinary temperatures and pressures.

12. A **vapor** is a gas that remains such only when heat is applied, and condenses into liquid or becomes solid at ordinary temperatures and pressures; steam is a vapor.

PROPERTIES OF MATTER

13. **Properties of matter** are those features or characteristics belonging to it and making it apparent to the physical senses; the properties of matter are therefore the cause of the phenomena associated with it. *General properties* are those belonging to all substances, and *special properties* are those belonging to some substances only.

14. **General Properties.**—Among the general properties are the following:

1. **Extension,** or *magnitude,* is the property of occupying space. Every material body has three dimensions: *length, breadth,* and *thickness;* the product of these three dimensions is its *volume.*

2. **Impenetrability** is the property making it impossible for two bodies to occupy the same space at the same time.

3. **Weight** is the measure of the earth's attraction for a body. All bodies, even gases, have weight. Weight is measured by comparison with some standard.

4. **Divisibility** renders any material body capable of being separated into parts.

5. **Porosity** is the name used to denote the fact that all substances are full of minute spaces, or *pores;* these pores differ

in size in different substances. By means of heavy pressure, water has been forced through plates of iron and of gold, thus proving that even these dense substances are porous.

6. **Compressibility** renders bodies capable of being diminished in volume by the application of pressure. The compressibility of a body is due to its porosity, the size of the pores being reduced in size under pressure. Gases are the most compressible substances and solids the least. In general, liquids and solids can be compressed very little.

7. **Expansibility** denotes the fact that under some conditions, when heated for example, the molecules of a substance move farther apart, increasing the size of the pores and the volume of a body of the substance.

8. **Elasticity** enables a body to return to its original form after conditions causing distortion have been removed. Rubber, glass, ivory, and steel are very elastic; clay and putty have very little elasticity.

9. **Inertia** renders a body incapable of changing its own condition of rest or of motion. Any material body at rest will remain so forever unless some form of energy sets it in motion, and then it would continue moving in a straight line forever unless acted on by some force outside of the body. For example, a baseball does not fly through the air until it is thrown or batted, and then it would never stop if the air did not resist its passage and if its weight did not cause it to fall to the earth.

10. **Mobility** means either of two things: that the location of a body in space may be changed or that the molecules of a body move with reference to each other, as in liquids and gases. The first is a general property common to all substances, and the second a special property belonging to certain substances only.

11. **Indestructibility** is a general property, since the total quantity of matter in the universe remains constant, as is explained in Art. **4.**

15. Special Properties.—Special properties of matter not possessed by all substances include *hardness*, *brittleness*, *tenacity*, *malleability*, *ductility*, *viscosity*, etc. For example,

diamonds are hard; glass is brittle; steel is tenacious because hard to pull apart; steel is also ductile because it can be drawn out into wire; gold is malleable because it can be hammered into thin sheets; tar at ordinary temperatures is viscous because it will not flow readily.

GRAVITATION AND GRAVITY

16. **Gravitation** is the name given to the mutual attrac tion that holds all the heavenly bodies, such as the stars, the sun, the earth, and the moon, in their proper paths, or *orbits*. **Gravity** is the attraction that holds bodies to the earth; the measure of gravity is *weight*. When a piece of lead is held aloft, gravity pulls it toward the earth so hard that the lead is called heavy. This attraction between the earth and a body on or near it is not the same at all points on the earth's surface, but for all practical purposes may be considered the same. It is greatest at the level of the sea, and least at points highest above this level, as on high mountains, or farthest below, as in deep wells or in mines.

17. The **center of gravity** of a body is the point at which the body is perfectly balanced, or the point at which the whole weight of the body may be considered as concentrated. The action of gravity makes each body take a position such that its center of gravity is as near the earth as it can get.

For example, a body suspended at a single point always swings or tilts until its center of gravity is as low as possible. The heavy side of a floating body always turns down; that is, the center of gravity seeks the lowest possible point.

18. A body is in a state of **equilibrium** when all the forces acting on it are balanced; such a body may be at rest or in motion, provided the motion is uniform. For example, a body moving at constant speed in a straight line is in equilibrium. Three states of equilibrium are generally recognized, namely, *stable*, *unstable*, and *neutral*.

A body is in a state of stable equilibrium when any small movement about its support raises its center of gravity so that

the body tends to return to its original position; for example, a body with a large flat base or with several points of support, as a brick lying on a large face, a table with four or more legs, a desk.

Unstable equilibrium is the condition when any movement of a body about its support gives the body a tendency to move farther, as is the case with a body that is balanced on an edge or a point.

Neutral equilibrium exists when movement of the body does not change the elevation of its center of gravity, as in rolling a ball or a cylinder on a level surface.

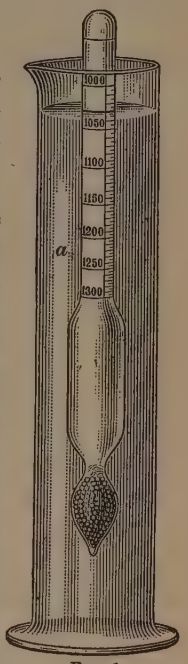


FIG. 1

19. The **specific gravity** of a substance is the quotient obtained by dividing the weight of a given volume of the substance by the weight of the same volume of some other substance used as a standard. Pure water at the temperature of greatest density (just above freezing point) is usually taken as the standard substance for solids and liquids, and air is the standard for gases. By some authorities water is the standard of comparison for solids, liquids, and gases. The specific gravity of gold is nearly 20, because a given volume of gold is nearly 20 times as heavy as the same volume of water.

20. For determining the specific gravity of liquids without weighing them, use is made of a **hydrometer,** Fig. 1. A glass tube *a*, one end of which forms a chamber that is weighted, or loaded, with shot or mercury (quicksilver), is placed in the liquid. The center of gravity being near the loaded end, the tube stands vertical. The point on the tube at the surface of pure water is marked 1,000 or 1.000 and a scale on the tube is marked above or below this point to indicate the specific gravity of other liquids; when placed in any liquid, the tube sinks to a point on the scale indicating the specific gravity. For example, if the mark 1275 is at the surface, the liquid is 1.275 times as heavy as water.

MOTION AND VELOCITY

21. **Motion** refers to change of location of a body with reference to some other body. A body may be simultaneously in motion with reference to some objects and at rest with reference to others; thus, passengers seated in a moving car are in motion with reference to objects outside the car and at rest with reference to objects inside the car.

22. The **path of a body in motion** is the line followed by its center of gravity. A hammer thrown through the air may turn and twist repeatedly, but the path of the body is traced by its center of gravity, usually within the head of the hammer.

23. **Velocity** is rate of motion, or the space traversed by a moving body in each unit of time; velocity is frequently stated as *speed*. Thus, sound travels in air at a velocity of approximately 1,000 feet per second, light and electricity travel at a velocity of approximately 190,000 miles per second, a railroad train may run at a speed of 40 miles per hour, etc. When equal distances are traversed in succeeding equal intervals of time, as every minute, the velocity is *uniform;* and when these distances are unequal the velocity is *variable*.

24. A **uniformly varying velocity** increases or decreases at a uniform rate during succeeding time intervals. For example, a body dropped from a balloon at a great height continually gains velocity as long as it falls freely. At the end of the first second, it is going at approximately 32.16 feet per second, at the end of the next second at 64.32 feet per second, at the end of the third second 96.48 feet per second, and so on, the gain of velocity each second being 32.16 feet.

25. **Acceleration** is the rate of increase, or the increment, of velocity. The acceleration of a freely falling body is 32.16 feet per second during each second, or as it is usually expressed, *32.16 feet per second per second*. In the case of a railroad train or an electric car, acceleration is usually stated in *miles per hour per second*, meaning the increase during each second in speed expressed in miles per hour.

26. The **total gain in velocity** of a body during a period of uniform acceleration is the product of its acceleration per second and the number of seconds, or

$$v = a\,t \qquad (1)$$

in which v = total gain of velocity;
a = acceleration, in feet per second per second;
t = time, in seconds.

If the body has a velocity v_1 (read *v sub 1*) at the beginning of the period and gains velocity at the rate of a feet per second per second, its velocity at the end of t seconds is

$$V = v_1 + a\,t \qquad (2)$$

For example, a body falling freely for 10 seconds gains velocity $v = 32.16 \times 10 = 321.6$ feet; and if it is started downwards at 100 feet per second, its velocity at the end of 10 seconds will be $V = 100 + 321.6 = 421.6$ feet.

27. The **average velocity** V_a (read *V sub a*) of a body during a period of uniform acceleration is the sum of its velocity at the beginning of the period and one-half of the gain of velocity during the period, or

$$V_a = v_1 + \frac{v}{2}$$

For example, if an electric car starts from rest, or 0 velocity, and accelerates at a uniform rate to a speed of 22 feet per second, its average velocity while accelerating is $V_a = 0 + \frac{22}{2} = 11$ feet per second. If the car was running at 10 feet per second at the beginning of the period of uniform acceleration, its gain of velocity was $22 - 10 = 12$ feet per second, and its average velocity while accelerating to a speed of 22 feet per second was $10 + \frac{12}{2} = 10 + 6 = 16$ feet.

28. The **distance** d traversed by a body during a period of time is the product of the average velocity during that time and the number of units of time. For example, an electric car traveling at an average velocity of 16 feet per second for

a period of 10 seconds moves through a distance $d = 16 \times 10 = 160$ feet.

29. If the distance that a body moves in a given time is known, its average velocity is the quotient of the distance divided by the time, or

$$V_a = \frac{d}{t}$$

Thus, a railroad train running 145 miles in 3.5 hours has an average speed $V_a = 145 \div 3.5 = 41.4$ miles per hour.

30. The **time required** to travel a given distance d at a uniform velocity V_a is found by dividing the distance by the velocity, or

$$t = \frac{d}{V_a}$$

Thus, light traveling at 190,000 miles per second, to traverse the 95,000,000 miles between the sun and the earth requires $t = 95{,}000{,}000 \div 190{,}000 = 500$ seconds.

31. **Retardation** is the rate of decrease of velocity; it is the reverse of acceleration. When a stone is thrown vertically upwards into the air, its velocity decreases uniformly to zero. If the stone starts upwards at 96.48 feet per second and ceases ascending in 3 seconds, its retardation is 32.16 feet per second per second. This would, in fact, be the case, since the stone would start and return at the same velocity and would ascend and return in the same time. If an electric car is brought from a speed of 12.5 miles per hour to rest in 10 seconds, the retardation, if uniform during the 10 seconds, is $12.5 \div 10 = 1.25$ miles per hour per second. In making calculations, therefore, retardation can be treated the same as acceleration.

FORCE

32. **Force** is applied energy; it is the action of energy in causing or tending to cause motion or change of motion of a body. Forces can be known and compared only by the effects they produce. The most familiar conception of a force is as a push or a pull. Force may exist without causing motion,

as when a man tries in vain to lift a heavy weight. The intensity with which a force acts may be constant or varying, giving rise to the names *constant force* and *varying force.*

Force may act through space without apparent connection with the object on which it acts. Thus, the force of gravity acts through space on all bodies near the earth; the force of magnetic attraction causes pieces of metal to move through space toward a magnet.

33. Forces have names according to their manifestations; *attractive force* is manifested in attraction and *repulsive force* in repulsion; *cohesive force*, known as *cohesion*, holds the molecules of a body together and causes its tenacity; *adhesive force*, or *adhesion*, causes unlike bodies to cling together, as mud adhering to the wheels of a vehicle; *accelerating force* causes *acceleration*, or increase of speed; etc.

34. **Stress** is a name indicating the force that tends to change the form of a body, or *strain* it. Thus, the stress on the wires attached to the top of a telegraph pole may cause a strain, or a bending of the pole. If the stress is removed from the wires, the elasticity of the pole may cause it to resume its upright position. If the pole remains bent after all the stress is removed, the stress was *beyond the elastic limit* and the strain that remains is called a *set.* The word strain is often wrongly used when stress is meant, as in speaking of the strain on a rope or a wire when the pull, or the stress, is meant.

35. **Measurement of forces,** in English-speaking countries, is usually expressed in pounds. Thus, the force with which a locomotive pulls or pushes a train of cars is expressed in pounds; the force with which electric wires are stretched between poles is expressed in pounds.

36. In physics, the word **mass** is used to designate the *quantity of matter* in a body. The size of a body does not determine the quantity of matter in it; for example, a bag of shot contains more matter than a bag of the same size filled with feathers. Nor does the weight of a body determine the quantity of matter in it; the weight of a body varies at different

places, as explained in Art. **16**, but the quantity of matter, or the mass, remains constant. In physical calculations, mass is considered as the quotient of the weight divided by the acceleration due to gravity.

Let m = mass;
w = weight, in pounds.

Then, $$m = \frac{w}{32.16}$$

For example, a body weighing 96.48 pounds has a mass $m = 96.48 \div 32.16 = 3$.

37. Whenever the motion of a body is increased or decreased or changed in any way, force must be exerted to overcome the inertia of the body, and this force, in pounds, is the product of the mass and the acceleration or retardation, in feet per second per second.

Let f = force, in pounds, to overcome inertia;
a = rate of acceleration or retardation, in feet per second per second.

Then, $$f = m\,a = \frac{w\,a}{32.16}$$

For example, the force necessary to accelerate or retard the velocity of a 90-pound body 5 feet per second per second when no other force is acting is $f = \frac{90 \times 5}{32.16} = 14$ pounds, nearly.

38. Every moving body meets with opposition tending to prevent, or retard, its motion. For example, a batted ball must force its way through the air; the wheels of a railroad car rub against the rails, and the axles rub in their bearings. The total force necessary to accelerate the speed of a body must overcome both inertia and all other opposing forces.

Let F = total force, in pounds;
f_1 = opposing forces other than inertia, in pounds.

Then, $$F = f + f_1 = \frac{w\,a}{32.16} + f_1$$

EXAMPLE.—What force is necessary to accelerate the speed of a 30,000-pound electric car 1.25 feet per second per second if the opposing force is 400 pounds?

SOLUTION.—According to the formula,

$$F = \frac{30{,}000 \times 1.25}{32.16} + 400 = 1{,}566 \text{ lb., approx.}$$

39. Centrifugal force is the name given the tendency of a revolving body to move away from the center about which it revolves. Thus, mud and water are thrown off the rims of rapidly moving vehicle wheels; a weight whirled rapidly at the end of a string seems to exert a strong pull on the string; heavy wheels, called *flywheels*, on machinery sometimes burst when rotated too rapidly, and pieces are thrown outwards with great force.

Centripetal force is the name given the force tending to draw a revolving body toward the center about which it revolves. Centripetal force is therefore a centering force, and its effect is opposite that of centrifugal force.

40. Representation of Forces.—In order to compare the effects of forces on bodies, the following conditions must be known for each force: (1) The point of application from which the force is acting; (2) the direction of action; and (3) the intensity of action.

These three conditions can be represented for each force by a single straight line, as at *A B*, Fig. 2. One end of the line, as *A*, represents the point of application, the arrowhead shows the direction of action, and the length of the line represents the intensity of action.

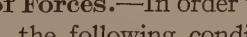


FIG. 2

When several forces are acting in different directions on a body, their combined effect is equivalent to that of a single force, called their *resultant*. The point of application, direction, and value of the resultant can be found if the three foregoing conditions are known for each force. The method of doing this, known as *composition of forces*, will be explained in a later Section.

WORK AND POWER

41. **Work** is the expenditure of energy in producing motion against an opposing force, as in lifting a body against the force of gravity. The unit of work most used in English-speaking countries is the **foot-pound,** which is the work done when a force of 1 pound is exerted continuously to produce motion through 1 foot of space, as in lifting 1 pound vertically 1 foot.

42. **Calculation of work,** in foot-pounds, always requires knowledge of the two factors, force, in pounds, and distance, in feet, through which the force produces motion. If the force is exerted without producing motion, as when a horse tries in vain to move a heavy load, no mechanical work is done; but if the load is moved a distance d, in feet, and a constant force of f pounds is exerted to move it, the work W, in foot-pounds, is

$$W = d\,f$$

For example, if a horse exerts a constant pull of 200 pounds while moving a load 5,000 feet, the work done is $W = 5{,}000 \times 200 = 1{,}000{,}000$ foot-pounds.

43. **Momentum** is quantity of motion. Every moving body has momentum, and the heavier the body or the more rapid its motion the greater is its momentum. When a rapidly moving bicycle strikes an obstruction, momentum may carry the rider over the handle bars. Momentum is measured by the product of mass m and velocity v. If the velocity is expressed in feet per second, the momentum will be in pounds, and as $m = \dfrac{w}{32.16}$,

$$momentum = m\,v = \frac{w\,v}{32.16}\ pounds$$

For example, if a car weighing 30,000 pounds is moving at a speed of 40 feet per second, it has a momentum of $\dfrac{30{,}000 \times 40}{32.16}$ $= 37{,}313$ pounds, approximately.

44. Power is rate of doing work, or the work done per unit of time. The work done in moving a body a given distance is the same whatever the time required, but the power depends on the time and is greatest when the work is done in the least time. The distinction between work and power should be remembered.

45. The **horsepower,** usually abbreviated H. P., is the common English unit of power; 1 horsepower equals 33,000 foot-pounds of work per minute, or 550 foot-pounds per second. Work is done at the rate of 1 horsepower when a body weighing 550 pounds is lifted vertically 1 foot in 1 second, or when 1 pound is lifted vertically 550 feet in 1 second; but if 60 seconds is taken to do the same work the rate is only $\frac{1}{60}$ horsepower. By formula, in which d and f have the meaning given in Art. **42,** t_m meaning time in minutes and t_s time in seconds,

$$\text{H. P.} = \frac{d\,f}{33{,}000\,t_m} \qquad (1)$$

$$\text{H. P.} = \frac{d\,f}{550\,t_s} \qquad (2)$$

Either formula can be used, according to whether time is given in minutes or in seconds.

EXAMPLE 1.—A load weighing 6,000 pounds is hoisted 600 feet in a mine shaft in $1\frac{1}{2}$ minutes. What is the rate of work in horsepower?

SOLUTION.—Here $d = 600$ ft., $f = 6{,}000$ lb., and $t_m = 1\frac{1}{2}$ min.; by formula **1,**

$$\text{H. P.} = \frac{600 \times 6000}{33000 \times 1\frac{1}{2}} = \frac{600 \times 2 \times 2}{11 \times 3} = \frac{800}{11} = 72\tfrac{8}{11}. \quad \text{Ans.}$$

EXAMPLE 2.—If an electric crane lifts a 1,650-pound motor 12 feet in 18 seconds, what is the rate of work in horsepower?

SOLUTION.—By formula **2,**

$$\text{H. P.} = \frac{12 \times 1650}{550 \times 18} = 2. \quad \text{Ans.}$$

FRICTION

46. **Friction** is resistance to motion due to contact of surfaces; thus, an effort, or force, is necessary to slide a book along a table top. The amount of friction depends on the pressure holding the surfaces together and on the nature of the surfaces. For example, ten times as much force is necessary to slide a 10-pound book as to slide a 1-pound book, provided the surfaces rubbing together are of the same nature; either book will slide easier on a smooth, polished table top than on a rough, unpolished surface.

47. The **coefficient of friction** is the quotient obtained by dividing the force necessary to move two surfaces upon each other by the perpendicular pressure holding them together, both the force and the pressure being expressed in the same units, usually pounds. For example, Fig. 3 represents a weight p resting on a horizontal plane and attached by a flexible cord to a weight f suspended by a cord that passes over a pulley a. If the pulley rotates without friction and the weights are so proportioned that the weight p will just move along the plane, the coefficient of friction is $\frac{f}{p}$. In this case, the perpendicular pressure holding the two surfaces together is the weight p and the force necessary to move them is the weight f.

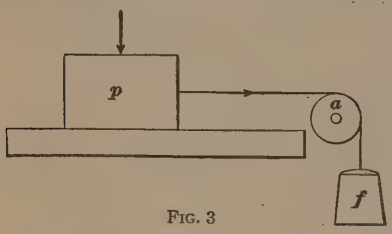


FIG. 3

48. Coefficients of friction differ for different materials and according to the lubrication of the surfaces. A general idea is given by the following, smooth surfaces being assumed in all cases:

Cast iron, wrought iron, steel, or bronze sliding on each other or each upon material of its own kind have coefficients of friction averaging .05 for good continuous lubrication,

.08 for medium-grade grease lubrication, .18 for surfaces only slightly greasy, and .25 for dry surfaces. The coefficient for leather on dry wood averages .47, on iron .28; for wood on dry wood .4 to .6, on soapy or wet wood .04 to .25; on stone .4; on metal .07 when well lubricated, .15 when soaped, .25 wet, and .4 dry.

49. The force necessary to move any two surfaces upon each other is the product of the perpendicular pressure holding them together and the coefficient of friction of the surfaces; the work done is this product times the distance moved. Thus, by formula,

$$f = c\,p \qquad (1)$$

$$W = c\,p\,d \qquad (2)$$

in which f = force, in pounds;
c = coefficient of friction;
p = perpendicular pressure, in pounds;
W = work, in foot-pounds;
d = distance, in feet, through which the force f acts.

EXAMPLE.—(*a*) Find the force necessary to drag a wet log weighing 750 pounds along a level wooden runway, if the coefficient of friction is .25. (*b*) Find the work done in dragging the log 60 feet.

SOLUTION.—(*a*) Since the runway is level, the weight of log is the perpendicular pressure and, when values are substituted in formula **1**,

$$f = .25 \times 750 = 187.5 \text{ lb.} \quad \text{Ans.}$$

(*b*) When values are substituted in formula **2**,

$$W = .25 \times 750 \times 60 = 11{,}250 \text{ ft.-lb.} \quad \text{Ans.}$$

50. The total friction between two bodies of wood, metal, or stone is independent of the extent of the surfaces in contact, provided that the lubrication is the same in all cases. For example, if a journal presses against its bearing with a force of 10,000 pounds and the coefficient of friction is .08, the total friction is $10{,}000 \times .08 = 800$ pounds, regardless of the size of the bearing. The larger the bearing, the less will be the pressure per square inch, and the less will be the wear. If the bearing is too small, however, the pressure per square inch will be so high that the lubricant will be forced out from

between the surfaces; excessive friction and heating may therefore occur until the bearing surfaces cling to each other, a condition called *seizing*, or *freezing*.

Friction is usually greater between surfaces of the same material than between surfaces of different material; the journals and bearings of machines are therefore usually made of different materials. Friction is greatest at the beginning of motion, which fact accounts in part for the greater effort required to start a machine than to keep it running; some of the increased effort necessary in starting is due to the inertia of the moving parts.

EXAMPLES FOR PRACTICE

1. If a railroad train runs 409 miles in $9\frac{3}{4}$ hours, of which $\frac{1}{2}$ hour is consumed in making 10 stops on the way, what is its average running speed?

Ans. 44.2 mi. per hr., approx.

2. An electric car weighing 40,000 pounds starts from rest and accelerates at 1.5 feet per second per second for 15 seconds. The track is straight and level and the coefficient of track friction is .0065. (*a*) What force is necessary for acceleration alone? (*b*) What force is necessary for both acceleration and friction? (*c*) What is the velocity, in feet per second, of the car at the end of 15 seconds? (*d*) What distance, in feet, is traversed during the 15 seconds? (*e*) What work is done during the period? (*f*) What is the momentum of the car at the end of the period? (*g*) What power is developed by the motors during acceleration?

Ans.
- (*a*) 1,866 lb., nearly
- (*b*) 2,126 lb., nearly
- (*c*) 22.5 ft. per sec.
- (*d*) 168.75 ft.
- (*e*) 358,763 ft.-lb., nearly
- (*f*) 27,985 lb., approx.
- (*g*) 43.5 H. P., nearly

MACHINE ELEMENTS

LEVERS

51. A **lever** is any rigid mechanical device having one fixed point, called a *fulcrum*, and acted on by two or more forces

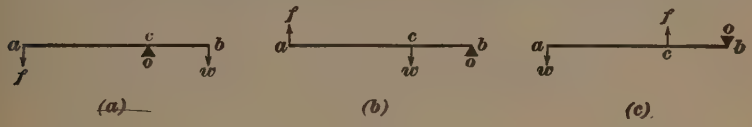


FIG. 4

at other points tending to turn the device on its fulcrum. Fig. 4 represents simple levers consisting of straight, rigid bars $a\,b$ each capable of being turned on its fulcrum o when acted on by unbalanced forces f and w. The fulcrum may be between the two forces, as in (a), or either force may be between the other force and the fulcrum, as in (b) and (c).

In Fig. 4, both forces are shown acting perpendicular to the lever, but either force may act in any other direction relative

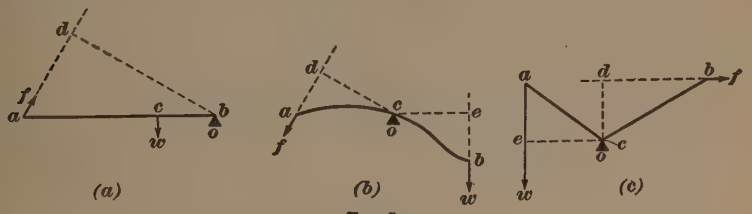


FIG. 5

to the lever, as in Fig. 5 (a). The lever may also be curved or bent, as in Fig. 5 (b) and (c).

52. The perpendicular distance, that is, the shortest distance between the direction in which a force acts on a lever and the fulcrum of the lever is called the **lever arm**, or simply the *arm*, of that force. Thus, the arm of force f in Fig. 4 (a)

is the distance $a\,c$, in view (b) the distance $a\,b$, and in view (c) the distance $c\,b$; the arm of force w is $b\,c$ in both (a) and (b) and $a\,b$ in (c). In Fig. 5, the arm of force f is $b\,d$ in (a) and $c\,d$ in (b) and (c); the arm of force w is $c\,b$ in (a) and $c\,e$ in (b) and (c).

53. When two forces acting upon a lever are balanced, the following principles will apply if the effect produced by the weights of the arms is neglected:

1. When the lever is at rest, *the product of one force and its arm equals the product of the other force and its arm.*

2. When the lever is in motion, *the product of one force and the distance through which it moves equals the product of the other force and the distance through which this force moves.*

EXAMPLE 1.—If a force of 1,000 pounds is pulling from a point 30 feet above ground on a pole, what opposite force to balance this pull must be applied at a point: (a) 15 feet above ground? (b) 21 feet above ground?

SOLUTION.—The pole forms a lever, and for the balanced condition assumed, the fulcrum is at the surface of the earth. In (a), the lever arms are 30 ft. and 15 ft., respectively; and in (b) they are 30 ft. and 21 ft.

(a) Let x = the unknown force to be found. According to principle 1, $x \times 15 = 1{,}000 \times 30 = 30{,}000$. If $15\ x = 30{,}000$, then

$$x = \frac{30{,}000}{15} = 2{,}000 \text{ lb.} \quad \text{Ans.}$$

(b) $x \times 21 = 1{,}000 \times 30 = 30{,}000$; therefore,

$$x = \frac{30{,}000}{21} = 1{,}428\tfrac{4}{7} \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—Wires attached to the top of a 36-foot pole have pulled the top 3 feet out of plumb. Workmen attach a rope to the pole 10 feet below the top and on digging away the earth supporting the base of the pole find that a horizontal pull of 1,600 pounds must be exerted on the rope to balance the pull of the wires. (a) If the pole turns about its lower end as a fulcrum, what is the force exerted by the wires? (b) If the constant force of 1,600 pounds draws the pole back to the vertical position, what is the distance traveled by the point at which the rope is attached?

SOLUTION.—(a) The pole forms a lever. The unknown force x acts 36 ft. from the fulcrum and the known force acts 26 ft. According to principle 1, $x \times 36 = 1{,}600 \times 26 = 41{,}600$. If $36\ x = 41{,}600$, then

$$x = \frac{41{,}600}{36} = \frac{10{,}400}{9} = 1{,}155.6 \text{ lb., approx.} \quad \text{Ans.}$$

(*b*) In this case, let x = the unknown distance to be found. According to the conditions given, the pull of the wires must remain constant at 1,155.6 lb., and the point at which this force acts must move 3 ft. According to principle 2, $1{,}600 \times x = 1{,}155.6 \times 3 = 3{,}466.8$. If $1{,}600\ x = 3{,}466.8$, then

$$x = \frac{3{,}466.8}{1{,}600} = 2.2 \text{ ft., approx.} \quad \text{Ans.}$$

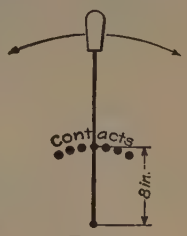


FIG. 6

EXAMPLE 3.—A certain device for controlling a motor is operated by means of a lever pivoted at one end and moved by a hand grasping the other end, as indicated in Fig. 6. The opposing force to be overcome in moving the lever is 10 pounds, caused by friction occurring at the contacts 8 inches from the pivot. (*a*) How long must the lever be in order that 4 pounds applied at the end will move it? (*b*) How far must the hand move in order to move the contacts 12 inches?

SOLUTION.—(*a*) Let x = the length of the lever. According to principle 1, $4x = 10 \times 8 = 80$. Therefore, $x = 20$ in. Ans.

(*b*) Let x_1 = the distance to be found. According to principle 2, $4\ x_1 = 10 \times 12 = 120$. Therefore, $x_1 = 30$ in. Ans.

54. A **compound lever** is a system of two or more simple levers arranged so that force applied to one is communicated through the whole system. Fig. 7 shows such a system with the levers lettered to correspond to the lettering in Figs. 4 and 5. If the forces are in equilibrium, downward force at f, Fig. 7, causes upward forces at c_1, a_2, and c_2 that balance the downward force at w. If the several lever arms are known,

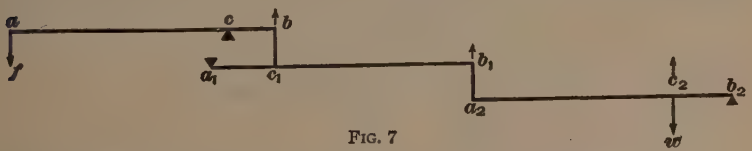


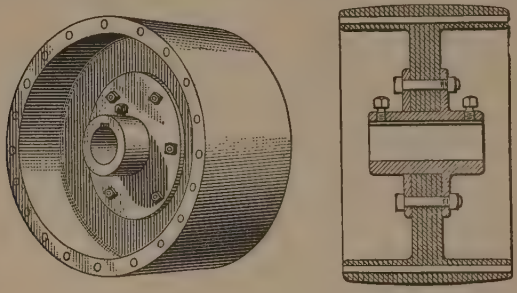
FIG. 7

the relative intensities of forces f and w for equilibrium can be calculated according to the principles stated in Art. **53.**

By means of suitable levers or systems of levers, a given available force can be much increased or decreased, as desired; but the actual work done will be very little changed. For example, a lever or a system of levers will enable one man to move a weight of several tons, but the movement will be very small compared with the distance through which his force is exerted.

PULLEYS

55. A **pulley** is a wheel with a smooth, flat or curved face and a central hub; the face is for carrying a belt and the hub



(*a*) FIG. 8 (*b*)

for mounting on a shaft. For use with electric generators and motors of capacities up to approximately 150 horsepower, pulleys made of paper are generally used; for larger electrical machines, the pulleys are usually of iron. Wooden pulleys are much used with other machines.

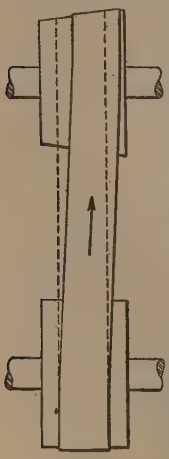


FIG. 9

In Fig. 8 (*a*) is illustrated a paper pulley, and in (*b*) is shown the method of construction. Disks and rings, punched from specially prepared paper sheets, are assembled and riveted solidly together, and an iron hub is bolted in the center. Iron and wooden pulleys are made both solid and split. A split pulley can be placed on a shaft at a point where it would be impossible to place a solid pulley

56. A **crowned-face pulley** has the form shown in Fig. 8 (*b*); such a face tends to hold a belt from running off either edge. A belt always climbs to the larger part of a pulley shaped as shown in Fig. 9 instead of running in the true position indicated by dotted lines. Pulleys for electric generators and motors usually have crowned faces, and the belts therefore tend to remain central on them.

57. Balancing Rotating Bodies.—Balancing a body that is to rotate is the process of changing the distribution of material so as to bring the center of gravity in the line about which the body rotates, this line being called the *axis of rotation*. Balancing is necessary to prevent the body from vibrating when it rotates.

58. A **standing balance** is given by mounting the body on an arbor, or piece of shaft, with protruding ends that rest on horizontal knife edges, so that the body can roll freely either way, as shown in end and face views of a pulley, Fig. 10. The arbor must be perfectly round and must fit closely in the hub

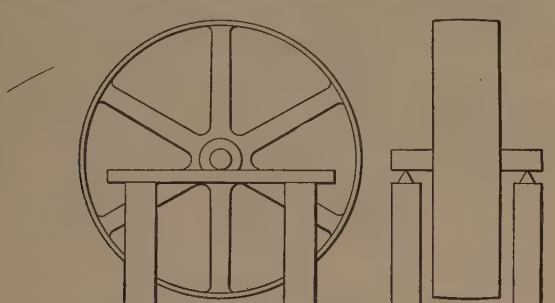


FIG. 10

of the body to be balanced. The knife edges are inverted **V**-shaped pieces of iron or steel planed true and carefully leveled.

When placed as shown in Fig. 10, the body rolls until the heavy side is down. Either some of the material is then removed from the heavy side or material is added to the light side until the body has no tendency to roll when placed in any position.

If given a good standing balance, narrow pulleys and all rotating bodies of similar shape will run fairly true and free from vibration at slow and moderate speeds; but wide bodies so balanced often vibrate badly, especially at high speed.

59. A **running balance** is needed for wide bodies and can be given only by equal distribution of material throughout the structure. This distribution is obtained by exercising

extreme care in construction. Vibration in pulleys that have a good standing balance is sometimes remedied by machining the inside of the pulley rim to run true with the outside.

PULLEY BLOCKS

60. A **pulley block** is a frame that carries one or more pulleys and is provided with supporting rings or hooks. The pulleys usually have grooved rims shaped to receive a rope and are frequently called *sheaves*. The word *tackle* is sometimes used as a general term in referring to a system of ropes, pulleys, and hooks for raising or lowering heavy weights; pulley blocks are therefore sometimes called *tackle blocks*. The

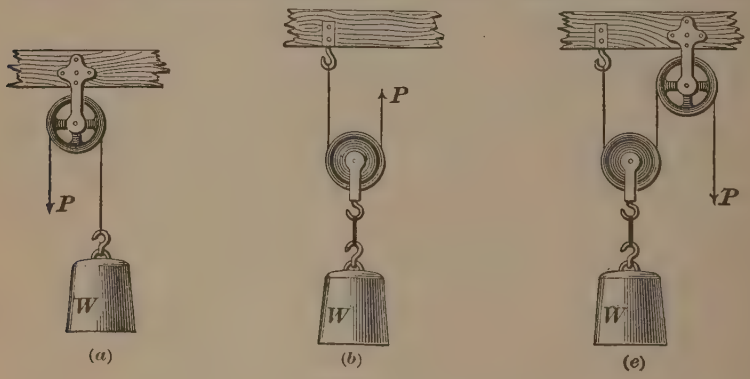


FIG. 11

expression *block and fall* is also used in referring to a pulley block and the rope.

61. A pulley block may be **fixed**, as in Fig. 11 (*a*), **movable**, as in (*b*), or two or more blocks may be used in a combination, as in (*c*). In these illustrations, the arrow at P indicates the direction of a force to support or raise a weight W. In (*a*), the weight is supported by one rope, and any point on the free end of the rope at P moves downwards as far as the weight moves upwards. Moreover, in order to produce equilibrium the force must equal the weight. A fixed pulley therefore serves merely to change the direction of a force.

62. With one movable pulley arranged as in Fig. 11 (*b*), the weight is supported by two ropes (strictly speaking, two parts of the same rope), each rope supporting one-half of the weight; and the upward movement of the free end of the rope at P must be twice that of the weight. This statement also applies to a combination such as is shown in (*c*); the fixed pulley serves only to change the direction of the force supporting the weight, and two ropes support the weight.

63. If friction and the weight of the movable block be neglected, the operation of all pulley blocks may be calculated according to the following general principles:

1. *For equilibrium, the force applied to the free end multiplied by the number of ropes supporting the movable block equals the weight supported by the movable block.*

2. *Any movement of the movable block multiplied by the number of ropes supporting it gives the movement of the free end of the rope.*

64. **Friction of moving parts** increases the force necessary at the free end of the rope in order to raise a given weight. To allow for friction, the theoretical force calculated according to principle 1, Art **63,** must be multiplied by a factor depending on the number of ropes supporting the movable block. The amount of friction varies with the conditions of the pulleys and ropes, but for most purposes fairly accurate results can be obtained by means of the following factors:

Number of ropes	2	3	4	5	6	7	8
Friction factor	1.18	1.25	1.33	1.43	1.52	1.59	1.67

65. The preceding may be expressed by the formula

$$f = \frac{e\,w}{n},$$

in which f = force, in pounds, applied to free end of rope;
e = friction factor;
w = opposing force or weight, in pounds;
n = number of ropes, or parts of rope, supporting w.

EXAMPLE 1.—How much force is required to raise 500 pounds with pulleys such as those shown in Fig. 11 (*b*) and (*c*)?

SOLUTION.—In either case, the number of supporting ropes n is 2 and the friction factor e is 1.18; therefore, the force

$$f=\frac{1.18\times 500}{2}=295 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—How much force is needed to raise 2,000 pounds with a set of blocks like that shown in Fig. 12?

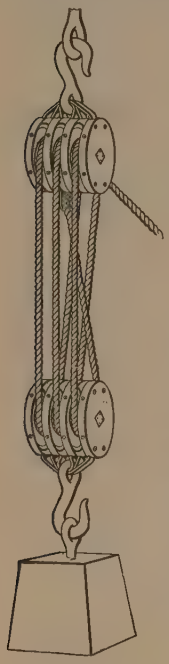


FIG. 12

SOLUTION.—The number of supporting ropes n is 6 and the friction factor e is 1.52. By the formula,

$$f=\frac{1.52\times 2{,}000}{6}=507 \text{ lb., approx.} \quad \text{Ans.}$$

66. To find the weight that can be raised by a given force use the formula

$$w=\frac{f\,n}{e},$$

in which the letters have the same meanings as those in the formula of Art. **65.**

EXAMPLE.—Find the weight that can be raised by 2 men pulling with forces of 125 pounds each on a set of pulley blocks having 4 supporting ropes for the movable block.

SOLUTION.—The friction factor e for 4 supporting ropes is 1.33, and the combined force f exerted by the two men is $2\times 125=250$ lb. When these values are substituted in the formula,

$$w=\frac{250\times 4}{1.33}=752 \text{ lb., nearly.} \quad \text{Ans.}$$

67. Chain Hoists.—In Fig. 13 is shown the **Weston differential chain hoist.** This hoist consists of two fixed pulleys and one movable pulley interconnected by an endless chain. The fixed pulleys differ in size, but are rigidly connected so that they must rotate together. The chain hangs from them in two loops, or *bights*, in one of which is the movable pulley; the other bight hangs free and one side of it is drawn downwards when operating the hoist.

68. Fig. 14 shows how the chain winds on the pulleys; the arrows indicate the directions in which the different parts move when the weight is raised by pulling downwards on the part *a* of the chain. The right side of the bight supporting the movable pulley moves down off the smaller of the two fixed pulleys, and the left side moves up on the larger pulley. Since the two fixed pulleys rotate together, the left side of this bight must move upwards faster than the right side moves downwards, and one-half the difference in these movements is the upward movement of the weight. The fraction *one-half* used here results from the principle of the movable pulley, as explained in Art. **62** in connection with Fig. 11 (*b*).

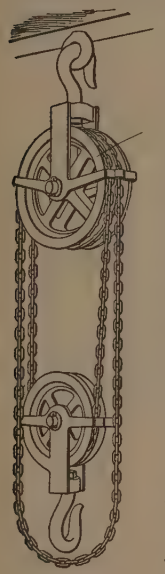


FIG. 13

With a differential chain hoist, a pull of 216 pounds drawing 30 feet of hand chain over its pulley will lift 1 ton 1 foot, and the load can be left suspended at any point without falling. The hoist is usually made in capacities up to 3 tons.

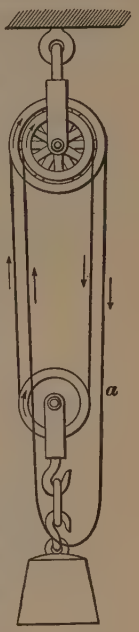


FIG. 14

69. In the **duplex-block hoist,** the force is transmitted from a hand chain to the load chain through a screw or a worm and worm-wheel (described later) and is made in sizes up to a capacity of 10 tons. A pull of about 87 pounds drawing 59 feet of the hand chain over its pulley will lift 1 ton 1 foot.

70. In the **triplex-block hoist,** the force is transmitted from the hand chain to the load chain through toothed wheels, or gears. This hoist is made in sizes up to a capacity of 20 tons. A pull of 82 pounds drawing 31 feet of the hand chain over its wheel will lift 1 ton 1 foot. The triplex hoist therefore gives greater increase of power than either the differential hoist or the duplex hoist.

EXAMPLES FOR PRACTICE

1. In a block and tackle with four parts of the rope supporting the load: (*a*) what force will be required to lift 300 pounds, assuming that there are no friction losses? (*b*) What force may be expected to be actually required, taking friction into account?

Ans. { (*a*) 75 lb.
(*b*) 100 lb.

2. A man weighing 120 pounds throws his whole weight on the free end of the rope of a block and tackle with six parts of the rope supporting the load. What weight can he raise: (*a*) if there are no friction losses? (*b*) friction is taken into account?

Ans. { (*a*) 720 lb.
(*b*) 474 lb., nearly

THE SCREW

71. The **screw** is a cylinder with a spiral *thread* winding around its surface, as in Fig. 15. For use in fastening parts of machines together, screws are practically always made of metal;

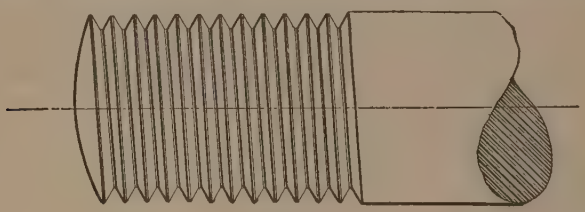


FIG. 15

they are known as *metal screws* if intended for screwing into metal and as *wood screws* if intended for screwing into wood.

72. Metal screws are made as *machine bolts*, *capscrews*, *stud bolts*, and *machine screws*.

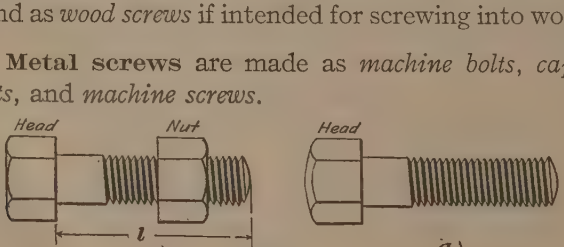


FIG. 16

A **machine bolt,** generally called a *bolt*, is shown in Fig. 16 (*a*); such bolts are regularly made in many sizes and with either hexagonal heads and nuts, as shown, or with square heads and nuts. The length is always measured under the

head, as indicated by the line *l*, and the diameter, or thickness, is measured directly through the center of the bolt between the tops of the threads.

A *capscrew*, shown in (*b*), is similar to a bolt, but is better finished and is usually supplied without a nut.

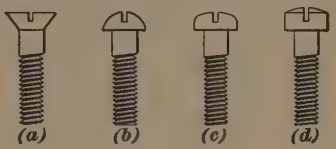


FIG. 17

A **stud bolt, or** *stud*, as it is usually called, is a bolt threaded at each end so that one end can be screwed into a fixed piece and the other can receive a nut.

Machine screws are small screws with heads slotted, as shown in Fig. 17, to receive the blade of a screwdriver. View (*a*) shows a *flat head*, (*b*) a *round head*, (*c*) a *button head*, and (*d*) a *fillister head*.

73. Wood screws are tapered to a point at the threaded end and are usually provided with slotted heads, which may be either flat or round. Large wood screws, called *lag-screws*, or *coachscrews*, are provided with square heads for turning with a wrench; they are much used to fasten cross-arm braces to poles for carrying electric wires.

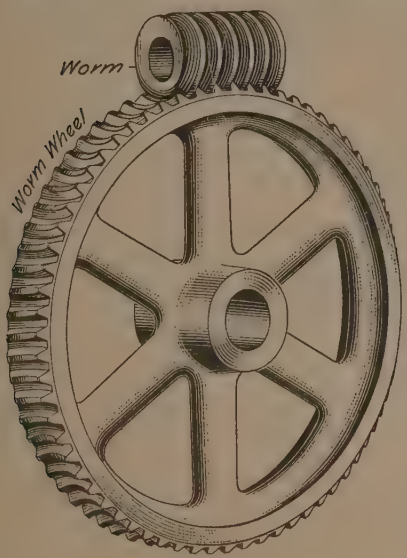


FIG. 18

74. The worm, Fig. 18, is a screw with threads that mesh with teeth on a worm-wheel. Each revolution of the worm moves the rim of the wheel a space equal to the distance between adjacent threads of the worm measured parallel to the worm-shaft. For each revolution of the worm-wheel a single-threaded worm, such as shown, must rotate as many times as there are teeth in the wheel.